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Comparisons of Urban Travel Forecasts Prepared with the Sequential Procedure and a Combined Model

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Abstract Detailed analyses and comparisons of urban travel forecasts prepared by applying the state-of-practice sequential procedure and the solution of a combined network equilibrium model are presented. The sequential procedure for solving the trip distribution, mode choice and assignment problems with feedback is the current practice in most transportation planning agencies, although its important limitations are well known. The solution of a combined model, in contrast, results from a single mathematical formulation, which ensures a well-converged and consistent result. Using a real network, several methods for solving the sequential procedure with feedback are compared to the solution of the combined model ESTRAUS. The results of these methods are shown to have various levels of instability. The paper concludes with a call for a new paradigm of travel forecasting practice based on an internally consistent model formulation that can be solved to a level of precision suitable for comparing alternative scenarios.

Keywords Combined network equilibrium models · Sequential procedure · Urban travel forecasting

Currently, the sequential, or four-step, procedure, (Ortúzar and Willumsen, 2001), is the most widely used approach to urban travel forecasting by transportation

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agencies throughout the world. Unfortunately, the sequential procedure has important weaknesses, particularly for congested networks; one of the main drawbacks is the inconsistency between values of generalized costs input to the trip distribution and mode choice steps, and the corresponding values output by the traffic assignment step. To overcome this problem, a method known as "solving the sequential procedure with feedback" was devised. Feedback consists of iterating the sequential procedure until the link flows, generalized costs, and corresponding destination and mode choices are similar from one iteration to the next. Even though these iterations reduce the inconsistencies to the some degree, important problems still persist. Boyce et al. (1994) gave examples for an aggregated network representing the Chicago region; however, their paper did not contain a detailed transit route choice model with vehicle capacities, as is applied here.

The need for a method that overcomes the above limitations is clear; many are available in the research literature. Beckmann et al. (1956), Evans (1973, 1976), Florian et al. (1975), Florian and Nguyen (1978), Frank (1978), Abdulaal and LeBlanc (1979), Aashtiani and Magnanti (1981), Sheffi (1985), Safwat and Magnanti (1988), Lam and Huang (1992) and Abrahamsson and Lundqvist (1999) each examined variants of what is known today as the combined network equilibrium model, which enables the steps of the sequential procedure to be solved consistently, thereby eliminating many of its inherent shortcomings. During the last two decades several combined models have been implemented that solve multiclass combined network equilibrium problems, including research by Lam and Huang (1994), Boyce and Bar-Gera (2001, 2003), De Cea and Fernandez (2001), De Cea et al. (2003); see Boyce and Bar-Gera (2004) for a recent review. Several comparative studies have been performed, showing the advantages of solving a combined model versus applying the sequential procedure; see Boyce et al. (1994), Barquín (1992) and Hasan and Safwat (2000). Finally, Williams and Lam (1991), Williams and Lai (1991) and Williams et al. (1991) have explored in detail the equilibration properties of the sequential procedure with respect to highway investment and multi-modal systems.

To provide more insight into this issue, in this paper we contrast these two methods by considering an example network for the city of Concepción, Chile. The combined network equilibrium model used is ESTRAUS (version 5.5.3.k), and the sequential model is VIVALDI. The latter embodies the same internal functions as ESTRAUS, but they are solved sequentially with feedback, which allows for an objective comparison of these methods.

1. Definition of Sequential and Combined Models

The travel forecasting procedures considered in this paper solve the trip distribution (destination choice), mode split (mode choice) and traffic assignment (route choice) models sequentially. That is, they receive as inputs the fixed trip productions and attractions for each zone in the network, and predict the trip matrices by mode, as well as vehicle and passenger volumes on links. For this study we utilize the specific functional forms shown in Table 1.

The sequential procedure solves these steps one after another, using outputs of one model as inputs to the next. This method, however, does not ensure consistency in the results among the steps. The combined network equilibrium

Table 1 Destination, mode and route choice models

Step	Model	Mathematical Formulation
Destination Choice	Gravity type model consistent with maximum entropy	$T_w = A_i O_i B_j D_j e^{-\beta L_w}$
Mode Choice	Discrete Choice Logit Model	$P_w^m = \frac{e^{-\gamma \cdot u_w^m}}{\sum_{m \in n} e^{-\gamma \cdot u_w^{m'}}}$
Route Choice	Deterministic user equilibrium (Wardrop)	$C_r - C_w^* \begin{cases} = 0 & if \quad h_r^* > 0 \\ \geq 0 & if \quad h_r^* = 0 \end{cases}$

Where:

w: Origin-destination zone pair (i,j).

T_w: Total trips between O/D pair w.

A_i, B_j: Balancing factors.

O_i: Total trips produced by zone i.

D_j: Total trips attracted by zone j.

L_w: Generalized travel cost for O/D pair w (logsum).

P_w^m: Probability of traveling on mode m for O/D pair w.

u_w^m: Utility of traveling on mode m for O/D pair w.

C_r: Generalized travel cost on route r for O/D pair w.

C_w^*: Minimum generalized cost for O/D pair w.

h_r^*: Equilibrium flow on route r for a O/D pair w.

β, γ: Calibration parameters.

model, in contrast, is a single mathematical formulation whose solution ensures consistency in the results. For details on the mathematical formulation and solution algorithm of the ESTRAUS combined model, see De Cea et al. (2003).

2. Description of the Experiments

The example network used to test the sequential procedure and combined model represents the city of Concepción, Chile. This network is a small-scale problem consisting of 219 zones, 573 nodes, and 2,217 links (road and access links). The demand and network data correspond to the calibration year 1998. The period modeled is the morning peak hour with a total of 210,846 trips considering six available modes (auto driver, auto passenger, bus, shared taxi, taxi and walk) and three trip purposes (work, study and other). Travelers are classified into nine categories defined on combinations of income level and car ownership rate. The structure of the demand model is described in Table 1, with a doubly constrained gravity model, and multinomial logit mode choice model, which includes each mode and user category.

The sequential procedure is solved by three different methods:

- One iteration of the sequential procedure (A)
- Several iterations of the sequential procedure using weighed link flows from the current and previously weighted iterations (with a fixed weight), as the starting point for the next iteration (B)

- Several iterations of the sequential procedure using weighed link flows from the current and previously weighted iterations (with variable weights), as the starting point for the next iteration; the variable weight is $1/k$ for the current solution and $(1-1/k)$ for the previous solution, where k is the iteration number, which is known as the Method of Successive Averages (MSA) (C).

The averaging of road link flows in methods B and C is performed following the assignment step in each feedback iteration. These new link flows in turn determine new link costs, which serve as the starting point in the trip distribution model of the next feedback iteration.

The number of feedback iterations typically performed by practitioners is three to five. In the studies reported here, we performed 30 iterations in order to examine the stability of the solution process. Since the calibration year data represents a low level of congestion, an increased demand scenario is created by multiplying the origin–destination (O/D) vectors by a factor to represent higher levels of congestion; e.g., a factor of 1.7 means the total demand is increased by 70%.

For the purpose of the comparisons reported here, ESTRAUS was solved to a higher level of precision than is customary for applications. Specifically, 200 diagonalization iterations were performed, which provides for average changes in link flows of no more than 0.02% and changes in O/D flows of no more than 0.01%.

The results of these three methods are compared with the combined model solution (ESTRAUS) for various levels of aggregation. The most general comparison shows differences in overall mode split, as well as the average travel times by mode (weighted average of all O/D pairs). More detailed analyses compare the travel times for each O/D pair in the network as well as auto flows on each link.

3. Main Results

The sequential procedure was solved with the three methods described above: A, B and C. For method B, the fixed weights tested were 0.3, 0.4, 0.6, 0.8 and 1.0. For example, a weight of 0.8 means that the current solution is weighed by 0.8 and the previously weighted solutions by 0.2 to obtain the new link flows for the next feedback iteration. Using a weight of 1.0 implies that only the current solution is used as a starting point for the next feedback iteration. We first present the results for the base year, and then show the effect of increased congestion in the network, such as might occur in a forecast year.

3.1. Base Year Analysis

The overall mode choices predicted by each feedback method at the regional level are essentially the same, and very similar to the mode choices predicted by the combined model, ESTRAUS: auto driver—8%; auto passenger—5%; bus—46%; shared taxi—5%; taxi—0%; and walk—36%. In order to analyze the differences among the feedback methods in more detail, in Fig. 1 we compare the stability in

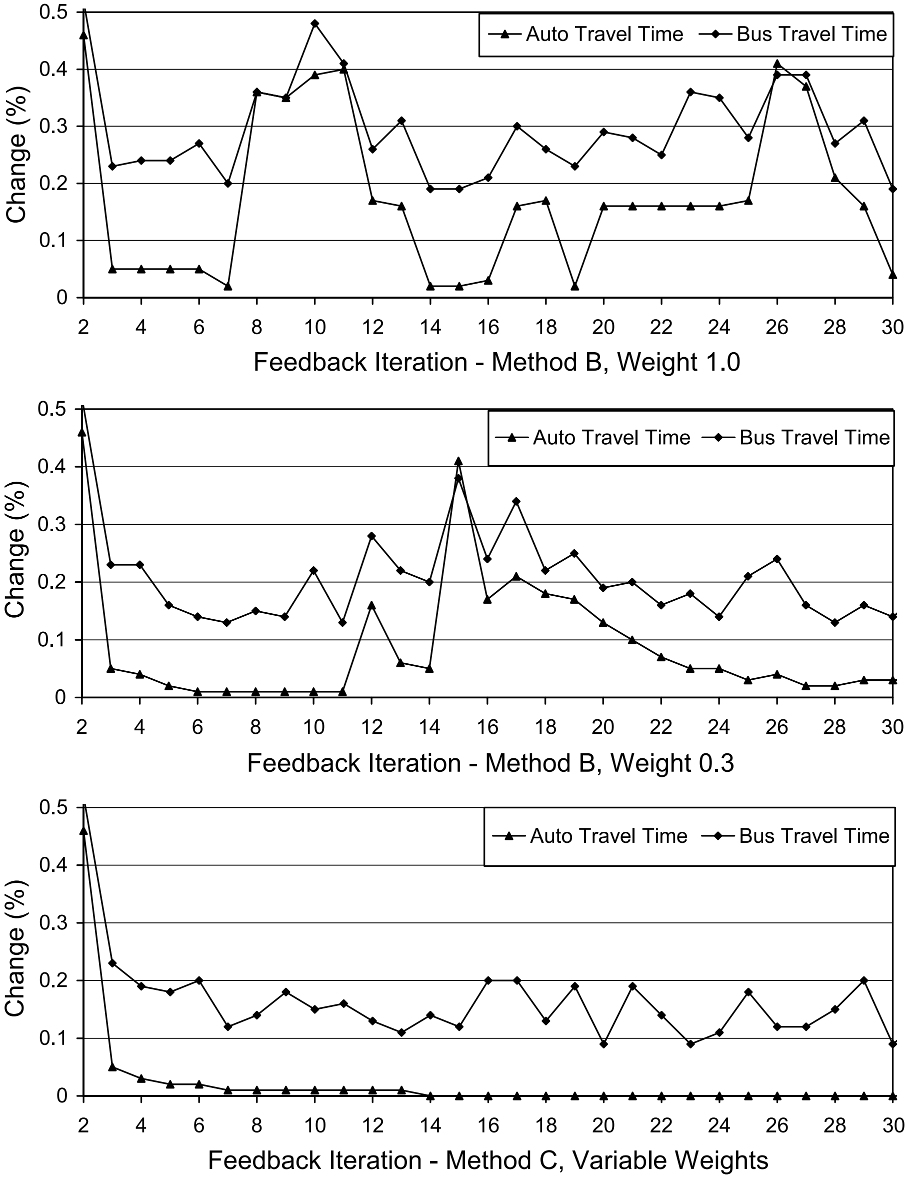

Fig. 1 Percent change in average O/D travel times for three versions of the Sequential Procedure

the travel times obtained from one feedback iteration to the next for the three methods. As can be seen, the feedback method that provides the most stable results in this case is the variable weight which decreases with the number of iterations (method C). The least stable solution is obtained for the method that uses the last solution as a starting point for the next (method B with a weight of 1.0). In performing further tests for other network conditions, method C was not always the most stable, but in all cases method B was the least stable.

Of course, a stable solution (method C in this case) is not necessarily a “good” solution. Specifically, successive solutions may not change either because they are near the correct solution, or because the differences are also forced towards zero, as is the case with the Method of Successive Averages as the weight approaches zero. Nevertheless, for the current analysis, method C was chosen as the “best” solution because in this case a more stable solution can be objectively compared to the true equilibrium solution (combined model). If the solution varies from one iteration to the next, a clear comparison cannot be done, since the comparisons will differ depending on which iteration is the last one.

As one can note in Fig. 1, the difference in bus travel times from one feedback iteration to the next is generally larger than the auto travel times. In the Concepción network, public transit modes show higher oscillations than private transportation modes because the network has a high level of transit usage (50% transit modal split in the morning peak period), and thus a higher level of in-vehicle congestion than for the road network. In Concepcion, 67% of travelers do not have auto modes available. As a result of this high level of in-vehicle congestion, which is explicitly modeled in the capacity-constrained transit assignment model, convergence is more difficult for the transit modes. From one iteration to the next, some travelers are changing from one transit mode to another (bus and shared taxi), as well as changing their optimal routes in the assignment step.

The variance in travel times would be even greater if only one iteration of the sequential procedure were performed (method A), which is not uncommon in current practice. To illustrate this point, note how the average travel times vary from the initial flow assumption (free-flow) to the solution resulting from solving the sequential procedure only once: auto—13%; bus—8%; shared taxi—0%;

Table 2 Average travel time (minutes) Sequential Procedures vs. ESTRAUS

Demand scenario 1.0

Mode	Feedback method (weight)				ESTRAUS
	B (0.3)	B (0.7)	B (1.0)	C	
Auto driver	12.7	12.8	12.7	12.7	12.6
Auto passenger	11.3	11.4	11.3	11.3	11.2
Taxi	8.1	8.1	8.1	8.1	7.8
Bus	18.0	17.9	17.9	18.0	17.4
Shared taxi	7.7	7.7	7.6	7.7	7.3

Demand scenario 1.7

Mode	Feedback method (weight)				ESTRAUS
	B (0.4)	B (0.8)	B (1.0)	C	
Auto-driver	17.3	17.2	17.1	17.2	15.5
Auto-passenger	15.6	15.4	15.3	15.4	13.9
Taxi	10.2	10.2	9.7	9.9	9.2
Bus	26.3	25.7	30.2	26.8	20.9
Shared taxi	11.0	8.7	6.8	10.3	9.1

taxi—13%. Clearly, using the sequential procedure without feedback (method A) produces results that are entirely inconsistent. Having shown this result, we do not return to method A in the remaining analyses.

Table 2 compares the average travel times obtained for each mode using the various feedback methods and ESTRAUS for the base scenario and the future scenario with 70% increased demand; the future scenario is discussed further in Section 3.2. Although the differences are small, note that these values are averages over the entire region. To analyze the differences among the methods in more detail, we next compare the travel times provided by each model by O/D pair.

Figure 2 compares bus travel times for each O/D pair using the sequential procedure with variable weights (method C) with the combined model ESTRAUS. For a discussion of Fig. 3, see Section 3.2 below. Even though the aggregate results in Table 2 are quite similar, at a more detailed level substantial differences can be noted. Figures 4 and 5 compare the auto flows on each link of the network resulting from feedback methods B (1.0) and C, respectively, with ESTRAUS. Again, differences can be noted. Results for method C appear to be slightly more similar to the combined model results.

In order to measure the level of error between the results provided by the sequential procedure and the combined model, we apply the commonly used measure, Root Mean Squared Error (RMSE), which has the following form:

$$\mathrm{RMSE} = \left[\sum_i \frac{(S_i - E_i)^2}{m}\right]^{1/2} \tag{1}$$

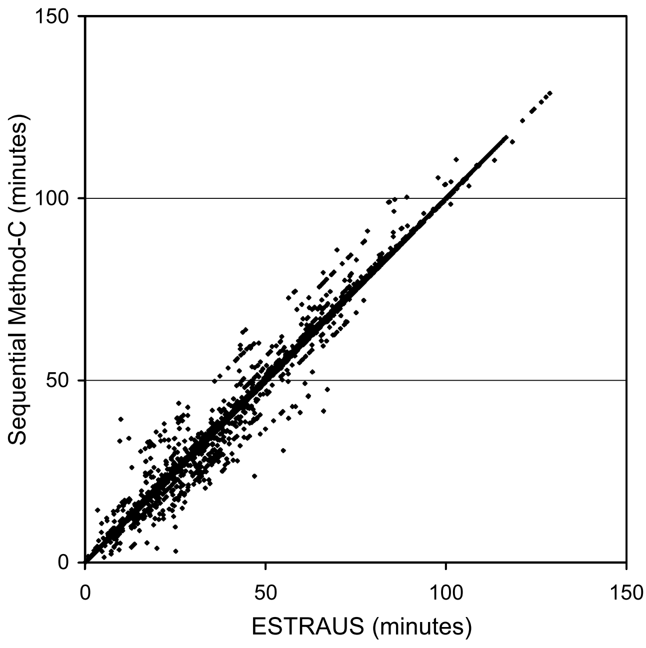

Fig. 2 O/D bus travel time—Sequential Procedure (method C) vs. ESTRAUS—Trips × 1.0

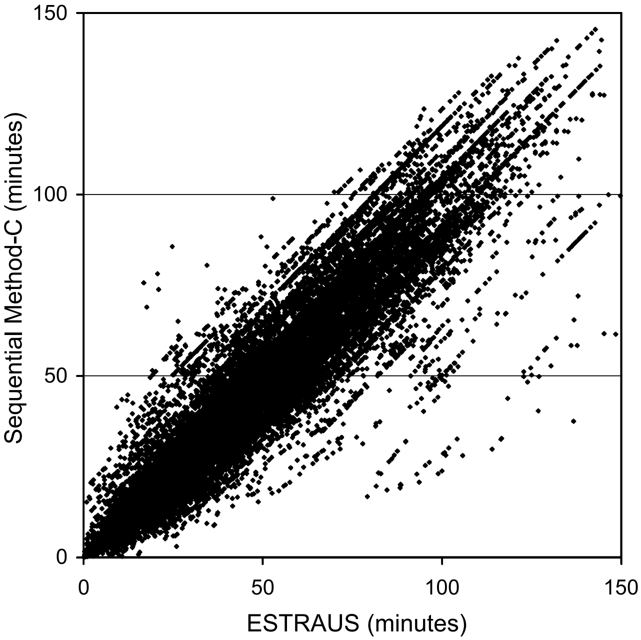

Fig. 3 O/D bus travel times—Sequential Procedure (method C) vs. ESTRAUS—Trips × 1.7

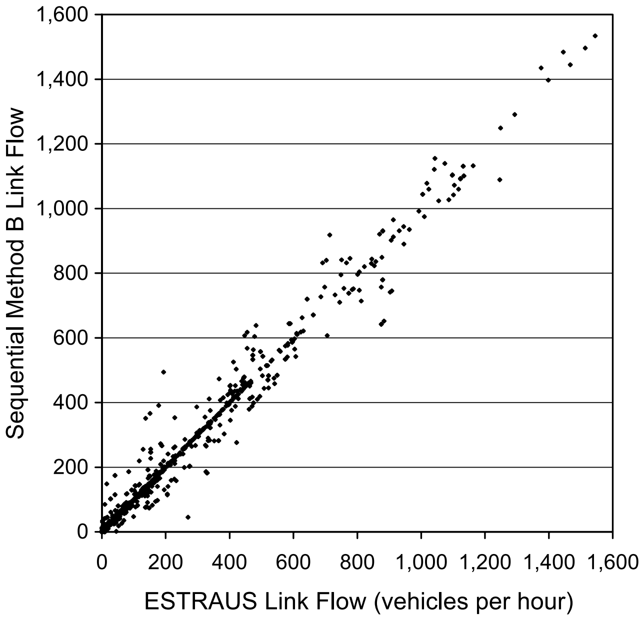

Fig. 4 Auto link flows—Sequential Procedure (method B) vs. ESTRAUS—Trips × 1.0

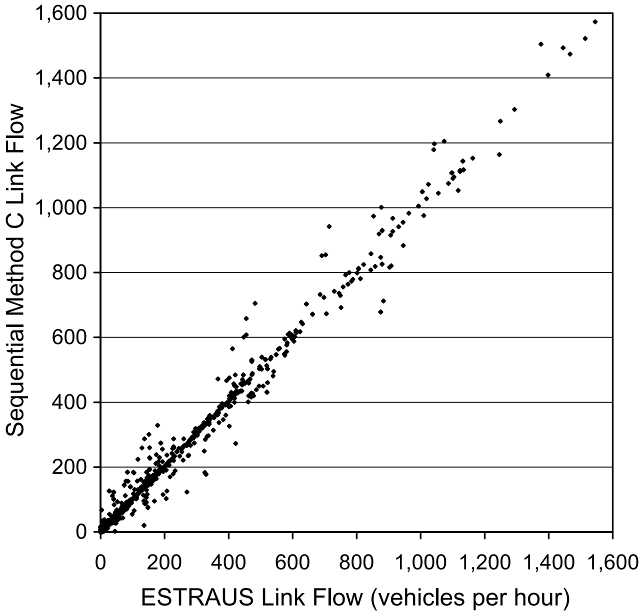

Fig. 5 Auto link flows—Sequential Procedure (method C) vs. ESTRAUS—Trips × 1.0

where *i* is the data element obtained (e.g., travel time, flow) that ranges from 1 to *m*, the number of cases; S_i is the result provided by the sequential procedure for case *i*; E_i is result provided by ESTRAUS for case *i*. If results between a given feedback method and the combined model are very similar, the RMSE should be near zero. The RSME for bus travel time is 1.1 min; for auto link flows, it is 20.5 vehicles/hour for feedback method C. Such a difference in RMSE is expected, since auto flows vary more widely.

3.2. Increased Demand Analysis

We first analyzed the effect of increased demand (attractions and productions) for 1.5 and 1.8 times the base values, and solved the sequential procedure with feedback (method B, weight 1.0). The differences in bus modal split from one feedback iteration to the next are shown in Fig. 6 for trip factors of 1.0, 1.5 and 1.8. As the demand increases in the network, the sequential procedure begins to oscillate from one feedback iteration to the next. This oscillation is produced at the most aggregate level possible (mode split), which implies the levels of service and flows from one iteration to the next of the sequential procedure have even greater differences. This oscillatory behavior is reduced by applying the other feedback methods. In this example network and demand scenario, the best results are obtained by using method C (variable weight decreasing with the number of iterations), as shown in the lower half of Fig. 6 for the increased demand factor 1.7.

Figure 7 compares the overall modal split obtained by the three methods of solving the sequential procedure with ESTRAUS for the increased demand factor

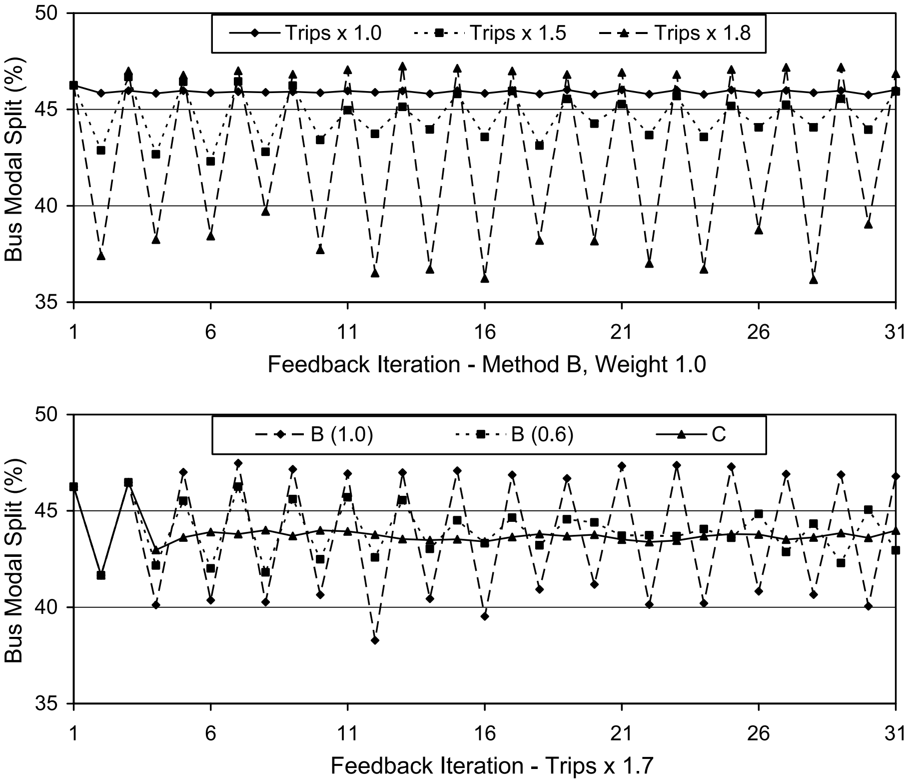

Fig. 6 Two studies of convergence of bus modal split

of 1.7; comparable results for the base case were given in the text in Section 3.1. These results vary from one feedback method to another. Clearly, performing feedback using the last solution as a starting point for the next is the least desirable (method B with weight 1.0). Particularly relevant, are the differences in

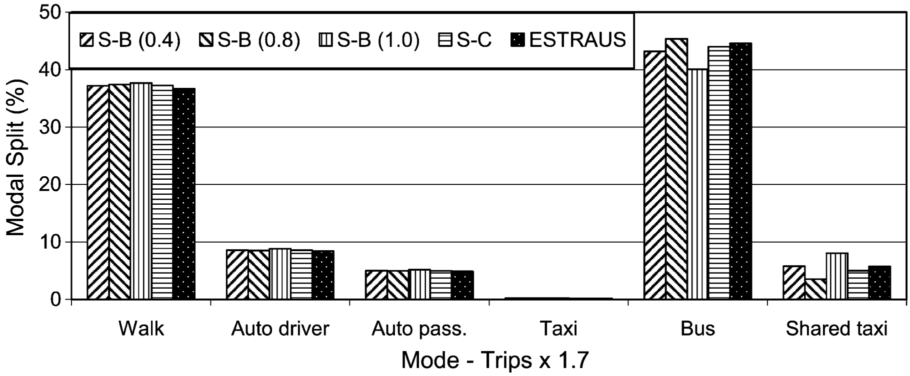

Fig. 7 Modal split for four solutions with the Sequential Procedure and ESTRAUS

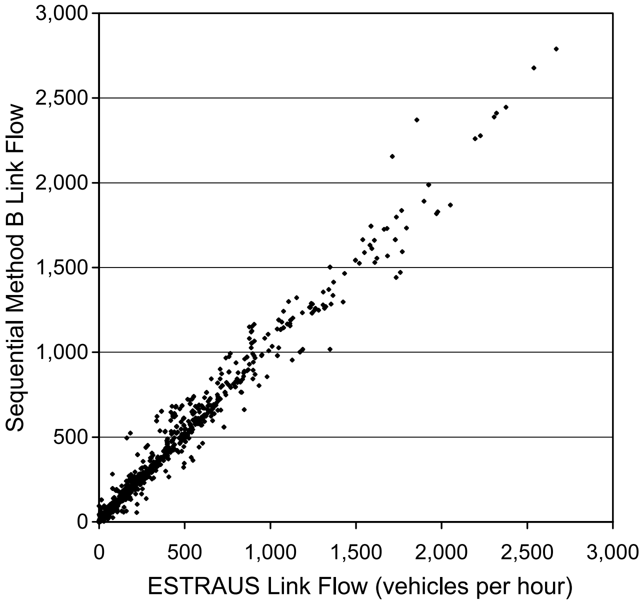

Fig. 8 Auto link flows—Sequential Procedure (method B) vs. ESTRAUS—Trips × 1.7

the bus and shared taxi mode shares. Theses differences are even more noticeable in the following detailed analysis.

Table 2, lower half, shows the average travel times for each mode using the three feedback methods with ESTRAUS for demand factor of 1.7; compare with the upper half of Table 2 for the base case. Notice the important differences in the

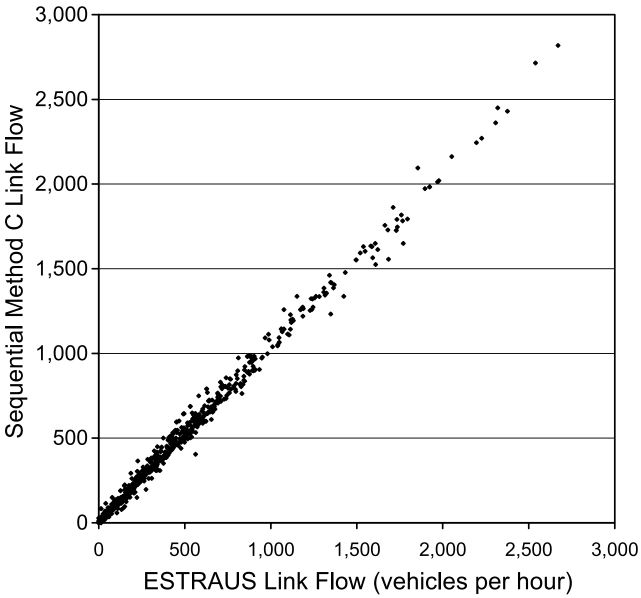

Fig. 9 Auto link flows—Sequential Procedure (method C) vs. ESTRAUS—Trips × 1.7

average travel times in the network. Although the results among the three feedback methods are quite similar, they are quite different from the results provided by ESTRAUS. For the auto driver mode, the difference in average travel times between the sequential procedure and the combined model are 11%; for the bus mode, the difference is 30%. Note these differences are for the regional level.

To analyze these differences in more detail, Fig. 3 shows bus travel times for each O/D pair for the sequential procedure (method C) versus the combined model for a demand factor of 1.7; compare with Fig. 2 for the base case. At this level of detail, the differences among the methods are much greater. Figures 8 and 9 compare the resulting auto flows on each link of the network for the sequential procedure, using method B with factor 1.0 and method C, respectively, with the results from ESTRAUS. The differences in results from the sequential procedure and the combined model are reduced by using feedback method C. Applying the RMSE measure to these results for bus travel times between O/D pairs and auto flows on links for the best feedback method C, we obtain values of 9.8 min for bus times, and 26.2 vehicles/hour for auto link flows. These errors are substantially increased as compared to errors obtained for the base case.

4. Conclusions

The main conclusions that can be drawn from the analysis comparing the traditional sequential procedure with a combined network equilibrium model for urban travel forecasting, can be summarized by the following points:

- If applied without feedback, the sequential procedure has important inconsistencies between the initial and final levels of service, such as auto and bus travel times. This effect is particularly strong if high levels of congestion exist in the network.
- Some feedback methods reduce these errors. Specifically in the cases tested, a variable weight which decreases with the number of iterations is shown to be the

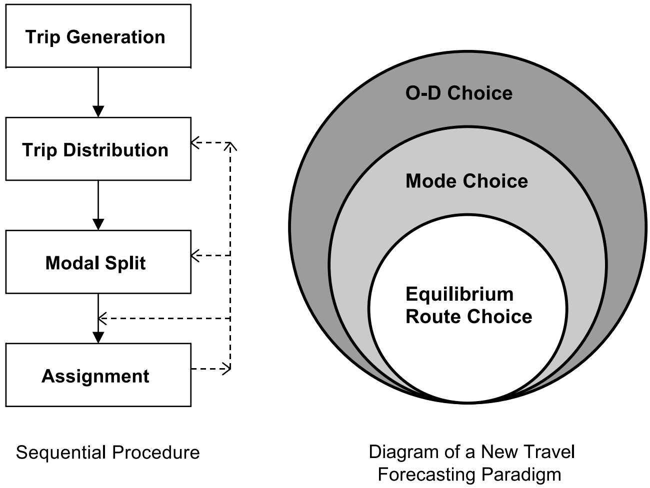

Fig. 10 Traditional and new paradigms of urban travel forecasting

best alternative. Improved feedback methods provide solutions that are similar to the solution of the combined network equilibrium model, which by definition provides the best consistency among levels of service.
- Even though improved feedback methods can be devised, important differences remain between these results and ones provided by a combined model.

The combined network equilibrium model, here exemplified by ESTRAUS, overcomes the main drawbacks of the sequential procedure. For this reason, a new conceptual paradigm of urban travel forecasting is advocated, which is graphically represented by the proposal of Boyce (2002), as shown in Fig. 10.

The solution of an internally consistent network equilibrium model allows the travel modeler to be sure that the model results are as precise and stable as possible. This approach does not imply that the results are necessarily accurate (i.e., replicate reality), which has to do with the quality of the input data and model calibration and validation, among others. Errors in forecasts should not be caused by using an inconsistent and unstable solution procedure, but may necessarily result from limitations beyond the control of the modeler. Finally, it is important to mention that this study is on-going. Tests with different types of networks and under different operating conditions are particularly important to analyze in the future.

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References

Abdulaal M, LeBlanc LJ (1979) Methods for combining modal split and equilibrium assignment models. Transp Sci 13:292–314

Abrahamsson T, Lundqvist L (1999) Formulation and estimation of combined network equilibrium models with applications to Stockholm. Transp Sci 33:80–100

Aashtiani HZ, Magnanti TL (1981) Equilibria on a congested transportation network. SIAM J Algebr Discrete Methods 2:213–226

Barquín M (1992) Análisis Comparativo de los Modelos de Transporte Secuencial y de Equilibrio Simultáneo. Memoria para optar al título de Ingeniero Civil. Pontificia Universidad Católica de Chile, Santiago, Chile

Beckmann M, McGuire CB, Winsten CB (1956) Studies in the economics of transportation. Yale University Press, New Haven

Boyce D (2002) Is the sequential travel forecasting paradigm counterproductive? ASCE J Urban Plann Dev 128:169–183

Boyce D, Bar-Gera H (2001) Network equilibrium models of travel choices with multiple classes. In: Lahr ML, Miller RE (eds) Regional science perspectives in economic analysis. Elsevier Science, Amsterdam, pp 85–98

Boyce D, Bar-Gera H (2003) Validation of urban travel forecasting models combining origin–destination, mode and route choices. J Reg Sci 43:517–540

Boyce D, Bar-Gera H (2004) Multiclass combined models for urban travel forecasting. Netw Spat Econ 4:115–124

Boyce D, Lupa M, Zhang Y (1994) Introducing feedback into four-step travel forecasting procedure versus equilibrium solution of combined model. Transp Res Rec 1443:65–74

De Cea J, Fernandez JE (2001) ESTRAUS: a simultaneous equilibrium model to analyze and evaluate multimodal urban transportation systems with multiple user classes. Proceedings of the Ninth World Conference on Transport Research. Seoul, Korea

De Cea J, Fernandez JE, Dekock V, Soto A, Friesz TL (2003) ESTRAUS: a computer package for solving supply–demand equilibrium problems on multimodal urban transportation

networks with multiple user classes. Presented at the Annual Meeting of the Transportation Research Board, Washington, District of Columbia
Evans SP (1973) Some applications of optimisation theory in transport planning. PhD thesis, Traffic Studies, University College London, London
Evans SP (1976) Derivation and analysis of some models for combining trip distribution and assignment. Transp Res 10:37–57
Florian M, Nguyen S (1978) A combined trip distribution, modal split and trip assignment model. Transp Res 12:241–246
Florian M, Nguyen S, Ferland J (1975) On the combined distribution-assignment of traffic. Transp Sci 9:43–53
Frank C (1978) A study of alternative approaches to combined trip distribution assignment modeling. PhD thesis, Regional Science, University of Pennsylvania, Philadelphia
Hasan MK, Safwat KNA (2000) Comparison of two transportation network equilibrium modeling approaches. ASCE J Transp Eng 126:35–40
Lam WHK, Huang H-J (1992) A combined trip distribution and assignment model for multiple user classes. Transp Res 26B:275–287
Lam WHK, Huang H-J (1994) Comparison of results of two models of transportation demand in Hong Kong: CDAM and a version of MicroTRIPS. J Adv Transp 28:107–126
Ortúzar J de D, Willumsen LG (2001) Modelling transport. Wiley, Chichester, UK
Safwat KNA, Magnanti TL (1988) A combined trip generation, trip distribution, modal split, and trip assignment model. Transp Sci 22:14–30
Sheffi Y (1985) Urban transportation networks: equilibrium analysis with mathematical programming methods. Prentice-Hall, Englewood Cliffs, New Jersey
Williams HCWL, Lam WM (1991) Transport policy appraisal with equilibrium models. I. Generated traffic and highway investment benefits. Transp Res 25B:253–279
Williams HCWL, Lai HS (1991) Transport policy appraisal with equilibrium models. II. Model dependence of highway investment benefits. Transp Res 25B:281–292
Williams HCWL, Lam WM, Austin J, Kim KS (1991) Transport policy appraisal with equilibrium models. III. Investment benefits in multi-modal systems. Transp Res 25B:293–316