



The **Travel** Model
Improvement
Program

Field Test of a Method for
Finding Consistent
Route Flows and Multiple-Class Link Flows
in Road Traffic Assignments

Helping Agencies Improve Their Planning Analysis Techniques

TMIP

Travel Model Improvement Program

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16. Abstract <p>Road traffic assignment, or forecasting route and link flows corresponding to fixed matrices of origin-destination (OD) flows by vehicle class on a road network for a given time period, is commonly applied by transportation planning practitioners. The standard user-equilibrium traffic assignment method uniquely determines the total flow on each network link, subject to convergence errors. Multiple-class link flows and route flows, however, are indeterminate. To ensure that route and multiple-class link flows are uniquely determined, or consistent, an additional assumption is required. One option is that proportions of flow over alternative route segments with equal costs are the same for all drivers, regardless of origin or destination. Analyses based on the assigned link and route flows by vehicle class, such as select link, select zone and emissions analyses, are often performed without considering this issue. Although such analyses have become important in practice, no commercial software system currently considers the indeterminacy of these flows.</p> <p>Traffic Assignment by Paired Alternative Segments (TAPAS) is a new algorithm offering the first practical way to address this issue. In this project six practitioners analyzed how route flows and/or multiple-class link flows generated by TAPAS compared with those found by their commercial software systems. A specialized tool VPAS was developed to compare the outputs of TAPAS and the practitioner software. The project team also undertook its own case study of the Chicago region with tools offered by four commercial software systems, which may be classified into two groups: link-based and quick-precision. Link-based tools applied in the project were CUBE, EMME and TransCAD; quick-precision tools applied were VISUM's route-based method and TransCAD's origin user-equilibrium (OUE) method. Findings of these applications may be summarized as follows:</p> <p>1. Select link results for link-based tools were approximately similar to those found by TAPAS; differences in flows through a selected link by OD pair were relatively small. However, small flows were observed in link-based solutions on non-equilibrium routes not found in more the precise TAPAS solutions. As a result, the number of OD pairs using a selected link was often much larger for link-based tools than for TAPAS. Analyses of flows on pairs of equal-cost segments showed that link-based solutions tended to satisfy approximately the proportionality condition. Slow convergence, however, is a costly limitation of link-based tools. Even so, the findings suggest that link-based tools do provide approximately proportional solutions, which was not realized before this project.</p> <p>2. Select link results for quick-precision tools were very different from those produced by TAPAS. In particular, where TAPAS predicted positive flows, quick-precision tools often gave zero flow from an OD pair through a selected link. Analyses of flows on pairs of equal-cost segments showed that quick-precision tools produced solutions that violate the proportionality condition. In two-class assignments for pairs of alternative segments, the proportions of flow found by quick-precision solutions were also different by class.</p>					
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Table of Contents

Technical Report Documentation Page	ii
List of Tables	v
List of Figures	vi
Preface	x
1. Introduction and Summary	1
1.1 Background	1
1.2 Benefits of consistent route flows	2
1.3 Project activities	3
1.4 Summary of project findings	3
2. Route Flows in Road Traffic Assignment: Principles and Algorithms	7
2.1 Route flow uniqueness and proportionality	7
2.2 Overview of algorithms	10
2.2.1 Tree-based algorithms	11
2.2.2 Bush-based algorithms	11
2.2.3 Traffic assignment by paired alternative segments	12
3. Implementation and Deployment	14
3.1 Implementation of TAPAS for use by the collaborators	14
3.2 Development and use of VPAS	14
3.2.1 Inputs	14
3.2.2 Loading	15
3.2.3 Validation	15
3.2.4 Select link analysis (SLA) and analysis of pairs of alternative segments (PAS)	17
3.2.5 Multiple-class verification and SLA/PAS	19
4. Collaborative Case Studies	20
4.1 Introduction	20
4.2 Maricopa Association of Governments case study	21
4.2.1 Background, collaborator and study objective	21
4.2.2 Description of case study findings	22
4.2.3 Conclusions of the collaborator	26
4.3 Pima Association of Governments case study	27
4.3.1 Background, collaborator and study objective	27
4.3.2 Description of case study findings	28
4.3.3 Conclusions of the collaborator	32
4.4 Delaware Valley Regional Planning Commission case study	33
4.4.1 Background, collaborator and study objective	33
4.4.2 Description of case study findings	33
4.4.3 Conclusions of the collaborator	37
4.5 Puget Sound Regional Council – PTV America case study	38
4.5.1 Background, collaborator and study objective	38
4.5.2 Description of case study findings	39
4.5.3 Conclusions of the collaborator	49

4.6	Puget Sound Regional Council – RST International case study	50
4.6.1	Background, collaborator and study objective	50
4.6.2	Description of case study findings	51
4.6.3	Conclusions of the collaborator	61
4.7	Ohio Department of Transportation case study	63
4.7.1	Background, collaborator and study objective	63
4.7.2	Description of case study findings	64
4.7.3	Conclusions of the collaborator	64
5.	Analyses of One-Class and Two-Class Flows for the Chicago Regional Network	65
5.1	Overview and objectives	65
5.2	One-class analyses	67
5.2.1	Select link analyses for North Avenue eastbound and Harlem Avenue southbound	67
5.2.2	Discussion of one-class select link analyses	72
5.2.3	Analysis of a pair of alternative segments for North Avenue	72
5.2.4	Discussion of one-class analysis of a pair of alternative segments	73
5.3	Two-class analyses	77
5.3.1	Select link analyses for North Avenue eastbound and Harlem Avenue southbound	77
5.3.2	Discussion of two-class select link analyses	77
5.3.3	Analysis of a pair of alternative segments for North Avenue	77
5.3.4	Discussion of two-class analysis of a pair of alternative segments	86
6.	Recommended Future Studies	87
	References	88

List of Tables

1.1	Summary of the case studies	3
2.1	Alternative route flow interpretations of the total link flows in Figure 2.1	7
2.2	Proportionality across origins	9
2.3	Classification of traffic assignment algorithms	10
4.1	Summary of the case studies	20
4.2	Convergence targets and computing environment	21
4.3	Computational time	26
4.4	Ratio of total segment flows for VISUM and TAPAS for Case 1	41
4.5	Ratio of total segment flows for VISUM and TAPAS for Case 2	45
4.6	Costs for 2030 by scenario and assignment algorithm	64
4.7	Present value of benefits by assignment algorithm	64
5.1	Overview of results presented in Chapter 5	66
5.2	Percentage of vehicle flow using each segment by class	86

List of Figures

2.1	Basic example of route flow uniqueness	7
2.2	Proportionality for multiple pairs of alternative segments	8
2.3	The TAPAS algorithm	13
3.1	VPAS interface	15
3.2	Scatter plot for link flow comparison	16
3.3	Cumulative log difference plot for link flow comparison	17
3.4	Scatter plot for select link analysis (OD) – linear scale	18
3.5	Cumulative log difference plot for select link analysis (OD)	18
3.6	Scatter plot for a pair of alternative segments – log scale	19
3.7	Log ratio plot for a pair of alternative segments	19
4.1a	East Thomas Road eastbound – linear scale	22
4.1b	East Thomas Road eastbound – log scale	22
4.2a	East Thomas Road westbound – linear scale	23
4.2b	East Thomas Road westbound – log scale	23
4.3a	Segment 1 for PAS analysis	24
4.3b	Segment 2 for PAS analysis	24
4.4a	Flows over Segments 1 and 2 (vehicles per period) – linear scale	25
4.4b	Flows over Segments 1 and 2 (vehicles per period) – log scale	25
4.4c	Log of the ratio of flows over segments 1 and 2	26
4.5	Pima Association of Governments road network	27
4.6a	Broadway – linear scale	28
4.6b	Broadway – log scale	28
4.7a	Grant Road – linear scale	29
4.7b	Grant Road – log scale	29
4.8a	Alvernon Way-Glenn Rd. (seg. 1) vs. Grant Rd.-Dodge Blvd. (seg. 2) – linear scale	30
4.8b	Alvernon Way-Glenn Rd. (seg. 1) vs. Grant Rd.-Dodge Blvd. (seg. 2) – log scale	30

4.9a	Orange Grove Rd. (segment 2) vs. Ina/Skyline Rd. (segment 1) – linear scale	31
4.9b	Orange Grove Rd. (segment 2) vs. Ina/Skyline Rd. (segment 1) – log scale	31
4.10	Location of the case study in central Philadelphia	33
4.11	Pair of alternative segments connecting node 5430 to zone 143	34
4.12a	Flow bundle for segment 1	34
4.12b	Flow bundle for segment 2	34
4.13	OD flows on segments 1 and 2 for TAPAS and VISUM (1E-3) – linear scale	35
4.14	OD flows on segments 1 and 2 for TAPAS and VISUM (1E-4) – linear scale	35
4.15	OD flows on segments 1 and 2 for TAPAS and VISUM (1E-3) – log scale	36
4.16	OD flows on Segments 1 and 2 for TAPAS and VISUM (1E-4) – log scale	36
4.17	Segments 1 and 2 linking node 2403 to node 1195	39
4.18a	SOV OD flows on segments 1 and 2 by TAPAS and VISUM – linear scale	40
4.18b	HOV OD flows on segments 1 and 2 for TAPAS and VISUM – linear scale	40
4.19a	SOV OD flows on segments 1 and 2 by TAPAS and VISUM – log scale	41
4.19b	HOV OD flows on segments 1 and 2 for TAPAS and VISUM – log scale	41
4.20a	Total link flows on links for route flows using segment 1 – linear scale	42
4.20b	Total link flows on links for route flows using segment 2 – linear scale	42
4.21a	Total link flows on links for route flows using segment 1 – log scale	43
4.21b	Total link flows on links for route flows using segment 2 – log scale	43
4.22	Segments 1 and 2 linking node 1173 to node 2968	44
4.23a	SOV OD flows on segments 1 and 2 by TAPAS and VISUM – linear scale	45
4.23b	HOV OD flows on segments 1 and 2 for TAPAS and VISUM – linear scale	45
4.24a	SOV OD flows on segments 1 and 2 by TAPAS and VISUM – log scale	46
4.24b	HOV OD flows on segments 1 and 2 by TAPAS and VISUM – log scale	46
4.25a	Total link flows on links for route flows using segment 1 – linear scale	47
4.25b	Total link flows on links for route flows using segment 2 – linear scale	47
4.26a	Total link flows on links for route flows using segment 1 – log scale	48
4.26b	Total link flows on links for route flows using segment 2 – log scale	48

4.27	Location of the three selected links on the I-5 Ship Canal Bridge	51
4.28	Northbound SOV OD flow, I-5 Ship Canal Bridge, regular lanes – linear scale	52
4.29	Southbound SOV OD flow, I-5 Ship Canal Bridge, regular lanes – linear scale	52
4.30	Northbound SOV OD flow, I-5 Ship Canal Bridge, regular lanes – log scale	53
4.31	Southbound SOV OD flow, I-5 Ship Canal Bridge, regular lanes – log scale	53
4.32	Northbound HOV OD flow, I-5 Ship Canal Bridge, regular lanes – linear scale	54
4.33	Southbound HOV OD flow, I-5 Ship Canal Bridge, regular lanes – linear scale	54
4.34	Northbound HOV OD flow, I-5 Ship Canal Bridge, regular lanes – log scale	55
4.35	Southbound HOV OD flow, I-5 Ship Canal Bridge, regular lanes – log scale	55
4.36	Northbound SOV OD flow, I-5 Ship Canal Bridge, express lanes – linear scale	56
4.37	Northbound HOV OD flow, I-5 Ship Canal Bridge, express lanes – linear scale	56
4.38	Northbound SOV OD flow, I-5 Ship Canal Bridge, express lanes – log scale	57
4.39	Northbound HOV OD flow, I-5 Ship Canal Bridge, express lanes – log scale	57
4.40	Northbound total link flow, I-5 Ship Canal Bridge, regular lanes – linear scale	58
4.41	Southbound total link flow, I-5 Ship Canal Bridge, regular lanes – linear scale	58
4.42	Northbound total link flow, I-5 Ship Canal Bridge, regular lanes – log scale	59
4.43	Southbound total link flow, I-5 Ship Canal Bridge, regular lanes – log scale	59
4.44	Northbound total link flow, I-5 Ship Canal Bridge, express lanes – linear scale	60
4.45	Northbound total link flow, I-5 Ship Canal Bridge, express lanes – log scale	60
4.46	I-5 Ship Canal Bridge express lanes northbound: select link flow comparison	62
5.1	North Avenue select link and pair of alternative segments	66
5.2	North Avenue eastbound OD flow (a-d)	68
5.3	Harlem Avenue southbound OD flow (a-d)	69

5.4 North Avenue eastbound link flow (a-b)	70
5.4 North Avenue eastbound link map (c-e)	71
5.5 Harlem Avenue southbound link flow (a-b)	70
5.5 Harlem Avenue southbound link map (c-e)	71
5.6 North Avenue PAS OD flow (a-b)	74
5.7 North Avenue PAS link flow (a-b)	75
5.7 North Avenue PAS link map (c-e)	76
5.8 North Avenue eastbound OD flow (a-b)	78
5.9 Harlem Avenue southbound OD flow (a-b)	79
5.10 North Avenue eastbound link flow (a-b)	80
5.10 North Avenue eastbound link map (c-e)	81
5.11 Harlem Avenue southbound link map (a-b)	80
5.11 Harlem Avenue southbound link map (c-e)	81
5.12 North Avenue PAS OD car flow (a-b)	82
5.13 North Avenue PAS OD truck flow (a-b)	83
5.14 North Avenue PAS total link flow (a-b)	84
5.14 North Avenue PAS link map (c-e)	85
5.15 North Avenue PAS total link flow (a-b)	84
5.15 North Avenue PAS link map (c-e)	85

Preface

From time to time over the past nine years, Hillel Bar-Gera sent me new traffic assignment codes for my experimental application and amusement. In December 2006, a new code called TAPAS arrived in my In Box. When I solved one of our large-scale test problems with this code, and compared it with earlier results, I realized that a dramatic change could be occurring. This code was really fast, and promised to find unique route flows during the same solution process, thereby solving a puzzle that Hillel had posed during his Ph.D. research nearly a decade earlier.

When Hillel told me he was planning to come to the US in 2008 for his sabbatical leave, I was motivated to find support for a program of field testing of his new algorithm. Eventually, this idea led to a cooperative agreement with the Federal Highway Administration in support of a field test of TAPAS by six collaborators recruited from planning agencies, consultants and the software development sector. In drafting the grant application, I consulted with my colleague, Marco Nie, concerning his interest in such an undertaking. He enthusiastically agreed to serve as the Principal Investigator, and has contributed generously of his time and expertise, especially with the development of a new tool, VPAS, that proved to be essential for comparing results from TAPAS with practitioner software systems.

Hillel, Marco and I are indebted to our professional colleagues in carrying out this field test. Some are old friends and former students; others are new friends made in the course of this project. First, we wish to thank our field test collaborators whose affiliations are found in Chapter 4: Rebekah Anderson, Chetan Joshi, Vladimir Livshits, Christopher Puchalsky, Aichong Sun and Robert Tung. Others who facilitated the process of recruiting these collaborators were Wolfgang Scherr, Robert Shull and Cherry Xiong, as well as Pitu Mirchandani and Mark Hickman, who introduced us to the metropolitan travel forecasting community in Arizona.

Second, we are indebted to our colleagues in the travel forecasting software development sector for their cooperation, encouragement and ongoing critical comments: Howard Slavin and Andres Rabinowicz of Caliper Corporation, Michael Clarke and Matthew Martimo of Citilabs, Michael Florian of INRO, and Klaus Nökel of PTV.

Third, we would like to acknowledge important contributions by two of my former students, Jeffrey Casello and Birat Pandey, who solved our Chicago test problems on software systems available to them, and thereby contributed to our assessment of TAPAS. Findings from their efforts are reported in Chapter 5.

Finally, we wish to acknowledge the contributions of our two junior co-authors, Yang Liu and Yucong Hu. Yang was essential to our effort to maintain order in our network data and experimental outputs as waves of results poured out from our collaborators. Yucong took on the task of deciphering the Ohio DOT model, and mastered TAPAS in the process.

Many others, too numerous to mention here, have contributed comments, questions and suggestions at conference and seminar presentations. We thank them for their encouragement.

March 2010

David Boyce

1. Introduction and Summary

1.1 Background

Road traffic assignment, that is forecasting of route and link flows corresponding to fixed matrices of vehicle origin-destination (OD) flows by vehicle class on a road network for a given time period is one of the most intensively applied tools utilized by transportation planning practitioners. Despite 50 years of development and deployment, first by public agencies and academic scholars, and later by commercial software developers, current travel forecasting software systems do not address several specific needs faced by practitioners:

1. consistent forecasts of multiple-class link flows within the solution for a single network, or across solutions for two or more alternative networks;
2. consistent forecasts of route flows by OD pair within a single network or across several networks;
3. consistent forecasts of route flows for use in traffic micro-simulation models.

What is meant by “consistent” flows? The standard user-equilibrium (UE) traffic assignment model applied in practice uniquely determines the *total flow* on each network link, subject to convergence errors. Multiple-class link flows and route flows, however, are not uniquely determined by the standard method. Therefore, additional assumptions are needed to assure that route flows and multiple-class link flows are uniquely determined in a single scenario, or are comparable across several alternative scenarios. Flows that are uniquely determined by the use of an additional assumption are termed *consistent*. One generally accepted method of determining consistent flows is to find the “most likely flows,” such as was proposed by Rossi, et al. (1989).¹

Why are “consistent” forecasts of link and route flows necessary and important?

First, consistent forecasts of *multiple-class link flows* are needed for the evaluation of proposed facilities for which vehicle classes are treated differently, such as toll roads, bridges, HOV lanes and HOT lanes, and for emissions and environmental justice analyses. However, class-specific link flows (trucks, HOVs, and SOVs by income class with differing travel behavior) are not uniquely determined by the standard user-equilibrium traffic assignment model deployed in currently available software systems. Consequently, a forecast of multiple-class link flows by such methods may be undetermined, and therefore could be misleading.

Second, consistent forecasts of *route flows* by OD pair are needed to apply methods such as select link analysis, select zone analysis, subarea analysis, evaluating components of generalized cost (time, distance, toll, etc.), which depend upon route flows, as well as modeling with license plate survey data, and OD estimation. Such methods are widely used by practitioners; however, as with multiple-class link flows, route flows are not uniquely determined by available methods.

¹ Another type of consistency in travel forecasts concerns the correspondence of impedances between the trip distribution and mode split steps and the assignment step; this type of consistency is not addressed in this report.

The focus of this report is deterministic user-equilibrium road traffic assignment, for which solution methods have improved substantially since the early 1970s following the recognition that the original user-equilibrium formulation of Beckmann et al. (1956) corresponded to attempts of early practitioners to assign trip matrices to road networks. Each of the commercial software systems now used in practice offers, or intends to offer in the near future, new options to solve the assignment problem to better levels of precision.

Assignment options, in common use for many years, allow one to utilize route flows and multiple-class link flows. Analyses based on the assigned link and route flows by vehicle class (e.g. emissions analyses, select link and select zone analyses), however, are often used without a full understanding of their validity, as noted in an EMME/2 manual (INRO, 1998, p. 4-357 and p. 6-24); see also the SATURN manual (Van Vliet, 2009, Section 7.1.6). Even so, such analyses have become increasingly important in practical applications.

Recently, Bar-Gera and Luzon (2007a, 2007b) examined the possible range of route flows that may comprise unique total link flows, observing for most OD pairs with multiple user-equilibrium routes that the route flows are effectively not determined. These route flows, in turn, are the basis for select link and select zone analyses. A similar indeterminacy likely applies to multiple-class link flows. Bar-Gera (2006, 2010) proposed a behavioral interpretation for the mathematical condition introduced by Rossi et al. (1989), which he characterized as an assumption of proportionality. This assumption states that *the proportions of flow on alternative route segments with equal costs are the same for all drivers*, regardless of their origins or destinations. By assuming that vehicle drivers traveling between all OD pairs behave similarly, the route flows are uniquely determined. Without this assumption, the proportions of flow on a pair of alternative route segments between one OD pair could be completely different from the proportions of flow on the same pair of route segments for another OD pair, which could render analyses based on these flows unuseable.

Traffic Assignment by Paired Alternative Segments, or TAPAS, a new algorithm stemming from the research of Bar-Gera (2010), offers the first practical opportunity to address these problems.

1.2 Benefits of consistent route flows

Three general types of improvements are expected from use of consistent route and multiple-class link flows: improved ability to forecast the users of selected links in a network; improved forecasts of the types of vehicles on links; and improved ability to segment vehicles by differences in driver behavior.

1. Improved understanding of the origin-destination movements contained in selected link flows allows transportation planners to estimate who benefits from link improvements so that transportation improvement programs can be better evaluated with respect to social equity and environmental justice criteria.
2. Consistent representation of the types of vehicles on links is important for predicting emissions, where multiple trip matrices correspond to various categories of vehicles with different emission characteristics. Similarly, assigning truck trip matrices by type separately

from private autos will yield consistent truck flows on links, which would not only improve goods movement analyses but allow better measurement of the consequences of heavy truck flows on links, such as their impacts on noise and safety.

3. Road pricing schemes are being considered by many MPOs and state DOTs. An important component in their evaluation is to determine who will pay the tolls, a task requiring consistent multiple-class link flows.

1.3 Project activities

This project centered around two activities:

1. Deployment of Bar-Gera’s TAPAS algorithm: Six practitioners used TAPAS, together with their practitioner software systems, to perform select link and related analyses. In each application, the practitioner determined, sometimes with the help of the project team, how the route flows and multiple-class link flows generated by TAPAS compared with those found by their practitioner software systems. A specialized tool VPAS was developed to compare the outputs of TAPAS and the practitioner software, and to validate that the assignments were compatible. Results of these comparisons of TAPAS and practitioner software are presented in Chapter 4, which comprise the principal project findings. The case studies, collaborators, number of user classes, software and method used are shown in Table 1.1.

Table 1.1 Summary of the case studies

<u>Case Study</u>	<u>Collaborator</u>	<u>Classes</u>	<u>Software</u>	<u>Method</u>
MAG	V. Livshits	1	TransCAD	Link-based
PAG	A. Sun	1	TransCAD	Link-based
DVRPC	C. Puchalsky	1	VISUM	Route-based
PSRC-PTV	C. Joshi	2	VISUM	Route-based
PSRC-RST	R. Tung	2	EMME	Link-based
ODOT	R. Anderson	2	CUBE	Link-based

2. Comparison of results among alternative software systems: To develop needed tools and to prepare for the efforts of practitioners, the project team undertook its own case study with each of the practitioner software systems using historical data files from the Chicago Area Transportation Study and OD matrices devised previously in their research. The correspondence, or the lack of it, of link and route flows over individual links or sequences of links (segments) were examined for each of the four practitioner software systems. The findings of this investigation are presented in Chapter 5.

1.4 Summary of project findings

Although the focus of this field test report is on route flows and two-class link flows, two important points should be stated at the outset to place the findings in proper context.

1. Comprehensive evaluation of software systems should address all of their features, an issue explicitly indicated in the report by MAG (Section 4.2.3, third conclusion), and discussed

informally by several other collaborators. Route flow proportionality is only one of many desired features of assignment. The TAPAS research code lacks many features commonly applied by practitioners in their activities, and is not directly comparable to commercial software systems. It is used here only to examine the issue of route flow proportionality.

2. Sufficient precision through improved convergence, and the computation time required to achieve it, are a major concern in practical applications of traffic assignment; see collaborator conclusions in Sections 4.2.3 and 4.7.3. Detailed evaluation of convergence performance is outside the scope of this research. Yet the project team believes that at the very least a distinction should be made between link-based assignment and quick-precision (route-based or origin-based) assignment, since the difference between these two groups of methods in terms of convergence performance is too great to be ignored.

All four software systems offer link-based assignment methods. At the outset of this project in 2008, quick-precision methods were offered by two software systems, TransCAD and VISUM. Such methods have since been implemented in EMME and CUBE, as well as a new tool in VISUM. In particular, a new route-based algorithm was released in EMME, version 3.2, in September 2009.

The collaborators applied in their case studies the four leading software systems in use in the US: CUBE, EMME, TransCAD and VISUM. Three single-class and three multiple-class case studies were conducted. Four case studies used link-based assignment tools based on the Frank-Wolfe (FW) method, also known as LA for linear approximation (TransCAD-FW, EMME-LA and CUBE-FW); the two other case studies used a quick-precision tool (VISUM). A single-class Chicago case study was solved by five commercial tools: three link-based (TransCAD-FW, EMME-LA and CUBE-FW), and two quick-precision (TransCAD-OUE, VISUM). A multiple-class Chicago case study was solved by two commercial tools: one link-based (CUBE-FW) and one quick-precision (VISUM). In addition, all case studies were solved by TAPAS.

Three main types of comparisons were conducted:

1. select link analysis, which is widely used in practice;
2. direct evaluation of proportionality by analysis of equal-cost pairs of alternative segments;
3. class-specific link and segment flows.

The findings led to similar conclusions regarding the three link-based tools and regarding the two commercial quick-precision tools.

Main findings regarding commercial link-based tools:

1. Select link analysis results of link-based tools are approximately similar to the TAPAS results (and to each other) in the sense that the differences in flows through the link by OD pair are relatively small. Similarly, small differences are observed for the so-called link-trace, i.e., the total flow on routes traversing other links and the selected link. However, small flows

can be observed in the link-based solutions on non-equilibrium routes, which are not found in the more precise TAPAS solutions. As a result, the number of OD pairs using a selected link is very different between TAPAS and the link-based tools, and quite different among the link-based tools themselves. In addition, link-based solutions use many more links in the link-trace compared to the TAPAS solution. These findings are shown for: two single-class collaborator case studies (Sections 4.2 and 4.3); one two-class collaborator case study (Section 4.6); the single-class Chicago case study (Section 5.2); and the two-class Chicago case study (Section 5.3).

2. Analyses of flows on equal-cost pairs of alternative segments show that link-based solutions in general tend to satisfy approximately the assumption of proportionality. This finding is based on: two single-class collaborator case studies (Sections 4.2 and 4.3); the single-class Chicago case study (Section 5.2); and the two-class Chicago case study (Section 5.3).
3. Slow convergence is a critical limitation of link-based tools. In a two-class collaborator case study, the TAPAS solution was found to converge substantially more precisely, and yielded estimates of benefits on the order of four percent lower than the less converged CUBE solution (Section 4.7). In a single-class collaborator case study, TransCAD-FW produced a less converged solution in a longer computation time than TAPAS (Section 4.2).

In summary, while slow convergence is costly, the findings available thus far suggest that link-based tools do provide approximately proportional solutions in terms of route flows and multiple-class link flows. This capability was not known at the outset of this project.

Main findings regarding commercial quick-precision tools:

1. Select link analyses for currently available quick-precision tools are quite different from those produced by TAPAS. The differences in OD flows through selected links are noticeable, particularly in cases where TAPAS predicts non-negligible flows, while the quick-precision tool predicts zero flow from the OD pair through the selected link. These findings are shown for: one single-class collaborator case study (Section 4.4); one two-class collaborator case study (Section 4.5); two tools (TransCAD-OUE and VISUM) in the single-class Chicago case study (Section 5.2); one tool (VISUM) in the two-class Chicago case study (Section 5.3). Differences are also noticeable in the link-traces of the selected links, as shown by the Chicago case studies (Sections 5.2 and 5.3).
2. Analyses of flows on equal-cost pairs of alternative segments show that quick-precision tools available commercially during the field test produce solutions that substantially violate the assumption of proportionality. The deviations are shown by maps of the link-traces of the segments for: one two-class collaborator case study (Section 4.5); and one tool (VISUM) in the single-class Chicago case study (Section 5.3). This finding is further supported by more extensive comparison of OD flows for: one single-class collaborator case study (Section 4.4); one two-class collaborator case study (Section 4.5); two tools (TransCAD-OUE and VISUM) in the single-class Chicago case study (Section 5.2); one tool (VISUM) in the two-class Chicago case study (Section 5.3).

3. The route-based method of VISUM often assigns all flow from each OD pair to one arbitrarily chosen segment from the two equal-cost alternatives.
4. In a two-class assignment for an equal-cost pair of alternative segments, the proportion of flow found by quick-precision tools can be very different by class (Section 4.5, Table 4.5).

In summary, while recognizing the advantage of quick-precision tools, caution is advised regarding analyses related to route flows and multiple-class link flows based on solutions produced by these methods.

Clarification: Upon receiving preliminary results from this project in April 2009, an option was added to TransCAD enabling the user to “Calculate Proportionality.” This feature was released for public use in September 2009, towards the end of this project. The TransCAD-OUE results in this report were produced by an earlier release, in which the new option to calculate proportionality was not implemented. The conclusions described above, and results presented in Section 5.2, pertain to the method that was applied.

Main findings regarding TAPAS:

1. When solutions are sufficiently converged, the assumption of proportionality is satisfied nearly perfectly, as shown by consideration of OD flows through selected pairs of segments, as well as with maps of link traces (Sections 5.2 and 5.3).
2. In two-class assignments for equal-cost pairs of alternative segments, the proportion of flow is the same for both classes (Section 4.5, Tables 4.4 and 4.5).

In summary, TAPAS is a quick-precision assignment tool providing route flows and two-class link flows that satisfy the assumption of proportionality.

Future research and testing could follow four main directions:

1. additional evaluation of the differences between route flow solutions in other types of analyses or in additional case studies with different features such as turning penalties, non-BPR volume-delay functions, etc.;
2. investigation of alternative approaches to satisfying route flow and multiple-class link flow proportionality in UE assignment;
3. studies of the structure of route sets and equal-cost pairs of alternative segments in practitioner networks;
4. empirical studies to test the validity of the assumption of proportionality in reality.

2. Route Flows in Road Traffic Assignment: Principles and Algorithms

2.1 Route flow uniqueness and proportionality

The main issue of uniqueness of route flows under the user-equilibrium (UE) assumption can be illustrated with a very simple example, shown in Figure 2.1. Suppose that the total link flows shown in the figure represent a perfect UE solution, for which identical travel times occur on a pair of alternative segments, [2, 3, 5] and [2, 4, 5]. The key question is how many travelers from each origin use each segment.

Three different solutions for the flow on each of the four routes in this network are given in Table 2.1. All three solutions correspond exactly to the same total link flows. If one wants to know how many travelers on link [2, 3] come from each origin, or if one extracts subarea data to study the weaving pattern on link [1, 2], the results of each route flow solution will lead to different answers. To choose among the infinite possibilities of route flow solutions for a single UE model, an additional behavioral assumption is required.

One plausible assumption is *proportionality*, namely that the proportion of travelers on each of the two alternative segments should be the same regardless of their origin or their destination. Indeed, this condition is observed in the solution marked by h^* in Table 2.1, as a ratio of 1 to 3 is found for routes R1 and R2 (25 to 75) as well as for routes R3 and R4 (15 to 45).

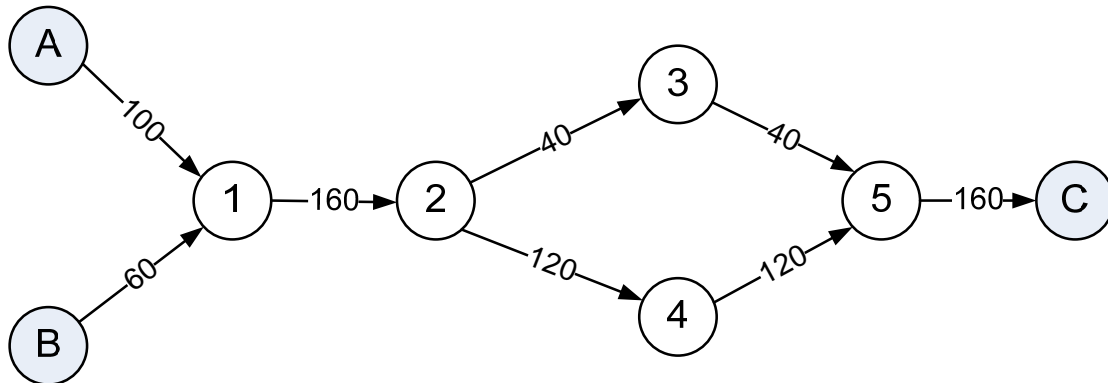


Figure 2.1 Basic example of route flow uniqueness

Table 2.1 Alternative route flow interpretations of the total link flows in Figure 2.1

OD	Route	Description	h^*	h_1	h_2
A-C	R1	A-1-2-3-5-C	25	40	0
A-C	R2	A-1-2-4-5-C	75	60	100
B-C	R3	B-1-2-3-5-C	15	0	40
B-C	R4	B-1-2-4-5-C	45	60	20

Proportionality is a fairly natural concept in a single class model, but it can also be used in multiple-class models. If travelers from origin B belong to two different classes, for example 40 passenger cars and 20 trucks, in passenger car equivalents, then there are two options. One

option is that only one class uses both alternatives. For example, link [2, 3] may be prohibited for truck use. In that case all trucks will use route R4 only; there is no need for an additional assumption on the way different classes use the two segments. Then, the assumption of proportionality will apply only to passenger cars from origins A and B. A related case occurs if the generalized cost of travel is equal on both segments for one class, but different for the other class. Since under the UE assumption travelers can use only the least cost route, this is in fact the same situation as the case of prohibited links, as far as determining route flows is concerned.

The other option is that both alternative segments have equal costs for both classes of travelers. In that situation the assumption of proportionality can be applied to class flows as well, enabling one to determine that there will be 10 passenger cars and 5 trucks on route R3, and 30 passenger cars and 15 trucks on route R4.

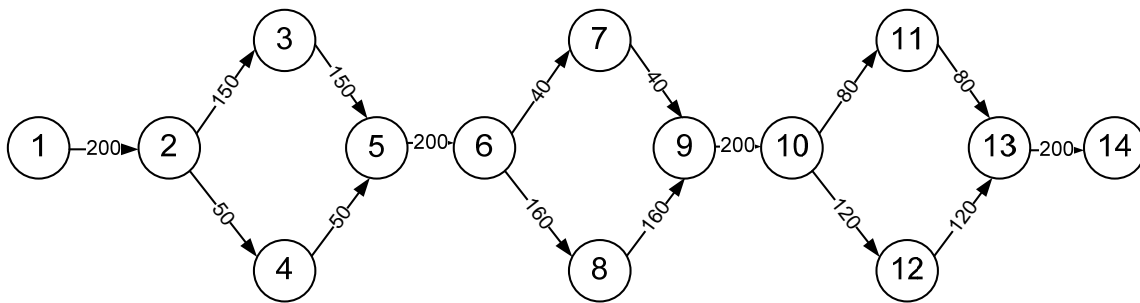


Figure 2.2 Proportionality for multiple pairs of alternative segments

In general networks there may be many pairs of alternative segments (PASs) with equal cost. For example, Figure 2.2 shows a network with three such pairs, resulting in eight different routes. The assumption of proportionality in this case simply means that the proportion of each route is the product of the proportions of its segments; for example, the proportion of the route flow using the top alternative in each pair, namely route [1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14], is $(150/200) \cdot (40/200) \cdot (80/200) = (0.75) \cdot (0.2) \cdot (0.4) = 0.06$. This proportion, multiplied by the total flow of 200, gives a total route flow of 12 vph.

Under the assumption of proportionality, all eight routes will be used. In general, proportionality implies a condition on the *set* of used routes that “no route is left behind.” Of course, this property does not mean that every route should be used, since under the UE assumption only least cost routes are used. So the assumption of proportionality requires that “no route should remain unused, unless there is a good reason.” Such a reason is that using the route causes a UE violation. A formal definition of proportionality, applicable to any network configuration, is discussed in: Bar-Gera and Boyce (1999); Bar-Gera (2006, 2010).

The assumption of proportionality was first introduced in Bar-Gera and Boyce (1999) as an interpretation for optimality conditions of route flow entropy maximization. Theoretically, one can construct examples where route flows are not uniquely determined even under the assumption of proportionality (Bar-Gera, 2006), whereas the entropy maximizing route flow solution is unique (Rossi et al., 1989). Thus, entropy maximizing implies proportionality, but it is a stricter condition. Bar-Gera (2006) showed that in realistic networks the difference between the

two conditions is relatively small; for example, in the case of the Chicago regional model, there are 90,723,930 degrees of freedom (algebraic dimensions in the space of route flows) that are not determined by the UE model, but only 91 of them remain undetermined once the assumption of proportionality is adopted. The practical significance of the difference between proportionality and entropy maximization has yet to be explored.

The task of identifying route flow solutions that satisfy the assumption of proportionality can be divided to two parts: within a single origin, and between origins. For a single origin, an explicit equation can be used to find route flows that satisfy within-origin proportionality. The equation states that the route proportion is the product of the origin-specific approach proportions of all merge nodes along the route; where approach proportion is defined as the ratio of origin-based link flow to origin-based node flow. In the example in Figure 2.2, merge nodes are 5, 9 and 13, and the computation is identical to the one presented above.

Proportionality across origins is evaluated by considering separately every pair of alternative segments shared by several origins. Table 2.2 shows hypothetical numerical values for flows on two alternative segment shared by origins 3, 25 and 42. In this case the proportion of flow on segment 1 is different for each origin. By adding segment flows from all origins we find an overall proportion of 0.8 for segment 1. If we apply this proportion to the flow from origin 3 summed over both segments, which is 25, we may expect segment flows from origin 3 to be 20 and 5. In this solution the flows are 15 and 10. Hence, the deviation from proportionality is 5 vph, which is in fact the maximum absolute deviation from proportionality over all origins for this specific PAS. In a general network the maximum absolute deviation from proportionality over all PASs can serve as a measure of the adherence to the assumption of proportionality.

Table 2.2 Proportionality across origins

Origin	Segment 1 flow	Segment 2 flow	Segment 1 proportion	Deviation
3	15	10	0.6	+5
25	15	0	1.0	-3
43	50	10	5/6	-2
Total	80	20	0.8	

If proportionality is not maintained across origins, adjustments are warranted. Considering the example in Table 2.2, if both segments are “isolated” in the sense that they do not contain any merging nodes for the relevant origins, then the deviations computed above can be used to determine the exact adjustment needed at the origin-based level to satisfy proportionality. The resulting flows will be 20 and 5 for origin 3; 12 and 3 for origin 25; and 48 and 12 for origin 43.

If the segments are not isolated, alternative adjustment schemes may be preferred (Bar-Gera, 2010). Either way, due to interactions between PASs, the adjustment of proportionality for one PAS may modify the origin-specific proportions of another PAS. Therefore, an iterative scheme appears to be needed, with proportionality adjustments made to all PASs in a cyclic manner.

Proportionality adjustments can be made in a post-processing step of any assignment algorithm, or they may be integrated into the algorithmic process. One potential advantage of within-process adjustments is the ability to identify an appropriate set of PASs, and a corresponding set

of routes. Additional studies are needed to compare post-processing and within-processing proportionality adjustment schemes.

2.2 Overview of algorithms

Traffic assignment algorithms can be categorized in various ways. A widely used scheme considers the aggregation level of the solution variables and the network to which the algorithms are applied. Traffic assignment traditionally operates on a set of shortest route trees, rooted either at an origin or a destination. These trees are generated iteratively using shortest route algorithms based on the current or “up-to-date” link costs, and stored explicitly or implicitly.

Another object, which was first used by Dial (1971) but whose utility in solving traffic assignment was not fully appreciated until the rediscovery of “origin-based” algorithms (Bar-Gera, 1999), is known as a *bush*, which can be roughly described as an acyclic subnetwork (i.e., a network with no cycles) rooted at an origin or destination. A bush enables the modeler to consolidate multiple shortest route trees as a single network object, recognizing that these trees often share a significant portion of their member links. Specifically, whenever a new shortest route tree is generated, only the links currently not on a bush need to be added. However, a bush is significant not only because it promises efficiency, but more importantly, because it offers a means to preserve *acyclicity*, a key feature of user-equilibrium and system-optimum flows (Newell, 1980).

The solution of the traffic assignment problem can be aggregated at different levels. At the most aggregated level, the assignment only stores and produces total flows for each link; algorithms for solving the problem at this level of aggregation are termed *link-based*. The total link flow solution is widely used because it is unique and often considered adequate for planning practice. However, there are circumstances where disaggregated flows on routes are desired. For example, a select link analysis, which decomposes total link flows by origin-destination (OD) pairs, is often used to examine the spatial impacts of a proposed road improvement.

Algorithms that directly solve the problem for route flows are called *route-based*. Route flows may also be obtained from link-based algorithms if the shortest route trees are retained. Moreover, route-based algorithms may be advantageous because they tend to converge faster than their link-based counterparts, especially at high levels of precision. In addition, disaggregating solutions for each origin (or destination) is considered a useful compromise between link- and route-based solutions, which leads to the origin-based approach. Table 2.3 shows a classification of research algorithms using these two criteria: the traditional tree-based algorithms aggregated at either the link or route level, and the bush-based algorithms that typically operate on origin or route flows. Traffic assignment algorithms applied in this project fall into one of these four categories, as detailed in the following section.

Table 2.3 Classification of traffic assignment algorithms

<u>Aggregation level</u>	<u>Link</u>	<u>Route</u>	<u>Origin/Destination</u>
Tree	Link-based	Route-based	
Bush		Algorithm B (Dial) TAPAS (Bar-Gera)	Origin-based (Bar-Gera)

2.2.1 Tree-based algorithms

The most widely used algorithm in this category is based on the algorithm proposed by Frank and Wolfe (1956). In each iteration, the algorithm solves a linearized sub-problem by assigning all trips between each OD pair to its current shortest route (all-or-nothing assignment). The solution from this subproblem provides a search direction along which the objective function may be improved by a one-dimensional line search. Although the Frank-Wolfe algorithm is easy to implement and requires modest computational resources to solve, its sub-linear convergence makes it very difficult to achieve the precise solutions that are often important for scenario comparison (Boyce et al., 2004; Slavin et al., 2009). Traffic assignment algorithms implemented in TRANSCAD, CUBE and EMME (linear approximation) all include the Frank-Wolfe method, although they differ in implementation details.

Route-based algorithms typically organize assignment by OD pair in the spirit of Gauss-Seidel decomposition. The assignment is cyclically decomposed for each OD pair in terms of route flows, and solved by equilibrating costs on existing routes. Various techniques have been used to equilibrate route costs. Often they involve shifting flows from more expensive routes to the least expensive one. A route-based algorithm needs to avoid route enumeration to be practically useful. In most previous studies, route enumeration is achieved by column generation, which iteratively generates routes only when they become required to solve the problem optimally (Dafermos, 1968; Gibert, 1968; Bertsekas, 1976; Bothner and Lutter, 1982; Jayakrishnan et al., 1994). Nevertheless, even storing and manipulating such a subset of routes can be expensive for large-scale networks used in practice today containing millions of OD pairs. Consequently, route-based algorithms are not widely available in current commercial software systems. An exception is VISUM, whose traffic assignment procedure is the only route-based algorithm included in this study. A route-based alternative algorithm was recently released for EMME as an addition to its linear approximation algorithm. This algorithm was not included in this project because it was not yet available to our collaborators.

2.2.2 Bush-based algorithms

Bush-based algorithms construct and maintain a bush for each origin, and restrict traffic assignment only to these bushes. They iterate between two sub-problems: *bush construction* (expanding or trimming bushes) and *bush equilibration* (finding equilibrium flows on bushes). According to the aggregation level, bush-based algorithms may be classified as *route flow* or *origin flow* based. In the former class is the algorithm of Dial (2006), which performs traffic assignment by swapping flows between the longest and shortest routes. Bar-Gera's origin-based algorithm (OBA) (Bar-Gera 1999, 2002) operates in the space of origin flows, which are represented by proportions of traffic arriving at each node from its predecessor links. In particular, Bar-Gera (2002) employed a projected quasi-Newton method to update origin flows. A variant of Dial's algorithm implemented in TRANSCAD 5.0 was included in this study. An algorithm, known as LUCE, has been implemented in VISUM, but was not ready for use in this project with regard to select link analysis.

Neither of the bush-based approaches explicitly stores routes flows. Therefore, in order to obtain route flows to support relevant applications, such as select link analysis, one has to impose

additional conditions, the most widely accepted condition being entropy maximization. As initially suggested by Bar-Gera (1999), the entropy condition can be applied to each origin individually. This method will generate unique route flows for a given bush-based solution. However, the bush-based solution itself is not necessarily unique unless the entropy condition is imposed *across* rather than *within* origins. Thus, neither Dial's nor Bar-Gera's origin-based algorithm is able to resolve the non-uniqueness of route flows.

As explained in Section 2.1, imposing an entropy condition across origins implies proportionality, namely that travelers distribute themselves in the same proportion on each of any two alternative equal cost route segments regardless of their origin or their destination. This observation forms the basis for Traffic Assignment by Paired Alternative Segments (TAPAS).

2.3 Traffic assignment by paired alternative segments

Traffic Assignment by Paired Alternative Segments (TAPAS) was developed to solve the static deterministic user-equilibrium traffic assignment in a computationally efficient manner while addressing the issue of route flow uniqueness by incorporating the assumption of proportionality (Bar-Gera, 2010). Being an iterative algorithm, TAPAS starts with an all-or-nothing solution, similar to many other algorithms. The main concept of the algorithm is the focus on *pairs of alternative segments* (PASs), achieved by storing a set of PASs, as well as a list of relevant origins for each PAS, which are updated throughout the algorithmic process. PASs are used for two purposes:

1. cost equilibration – by simply shifting flow from one segment to the other as needed;
2. proportionality adjustments – as discussed in Section 2.1.

A description of PAS set management procedures requires a fairly elaborate discussion, which is provided in Bar-Gera (2010). The overall algorithmic structure is shown below in Figure 2.3.

Find initial solution using all-or-nothing assignment
Repeat iteratively:
 For every origin
 Remove all cyclic flows
 Find tree of least cost routes
 For every link used by the origin which is not part of the tree
 If there is an existing effective PAS
 Make sure the origin is listed as relevant
 Else
 Construct a new PAS
 Choose a random subset of active PASs
 Shift flow within each chosen PAS
 For every active PAS
 Check if it should be eliminated
 Perform flow shift to equilibrate costs
 Redistribute flows between origins by the assumption of proportionality
Final proportionality iterations:
 For every active PAS
 Redistribute flows between origins by the assumption of proportionality

Figure 2.3 The TAPAS algorithm

3. Implementation and Deployment

3.1 Implementation of TAPAS for use by collaborators

The original implementation of TAPAS was designed and tested only on single-class problems. In practice, however, it is very common to perform multiple-class assignments. Therefore, the first adaptation for implementing TAPAS was to solve multiple-class assignment models. The structure of TAPAS is such that adding this feature did not require any major reorganization of the implementation, but several issues had to be addressed, including input and output procedures.

Because of the focus on global performance measures needed for research, the original implementation included very few options to output and analyze the results. The most important output option needed for this project was a feature for select link and select segment analyses. The preparation of a select link/segment tool needed to consider that flows are stored by origin, and not by route. Therefore, a route flow interpretation (Bar-Gera, 1999) needed to be applied in a computationally efficient manner. This tool also enables one to identify both the OD-trace and the link-trace of the selected link/segment.

3.2 Development and use of VPAS

VPAS was developed to compare the results of select link analyses and pair of alternative segments analyses for several practitioner software systems with TAPAS. In each application, several selected links or pairs of alternative segments were identified for study. VPAS was used to present the results in the form of plots to provide insights concerning the characteristics of TAPAS and the practitioner software. Details of how VPAS is applied are described in the following subsections; these user-oriented instructions are also helpful in understanding the resulting plots.

3.2.1 Inputs

Specify the directory where the input file used by TAPAS is stored for a given application by its “name”. VPAS requires two TAPAS input files: name_net.dat (network file) and name_trp.dat (OD file), where “name” is given by the user. In many cases, TAPAS and the practitioner SOFTWARE employ two different sets of IDs for nodes and links. In this case, a third file, name_nid_SOFTWARE.dat, provides a mapping between TAPAS node ID and the original node IDs from the SOFTWARE. Note that “SOFTWARE” should be replaced with the name of the software one is using in the software B drop-down list, such as TRANSCAD-FW, CUBE, etc. as shown in Figure 3.1. One name_nid file might be needed for each software to be compared. (Warning: VPAS could crash if this file is not provided when it is actually needed).

Input the coefficients of travel time, toll and distance in the defined travel cost function. If travel cost is equal to travel time, the coefficients are 1, 0 and 0 respectively.

3.2.2 Loading

Click “Load” to load the TAPAS project, which can require up to one minute depending on the network size and computer. Once loaded, one can check the network topology using the drop-down menu “project->display”. In order to do that, one needs to have “name_nod.dat” in the TAPAS folder, which specifies the node coordinates. One can also check TAPAS input files using “project->network” and “project->trip table”.

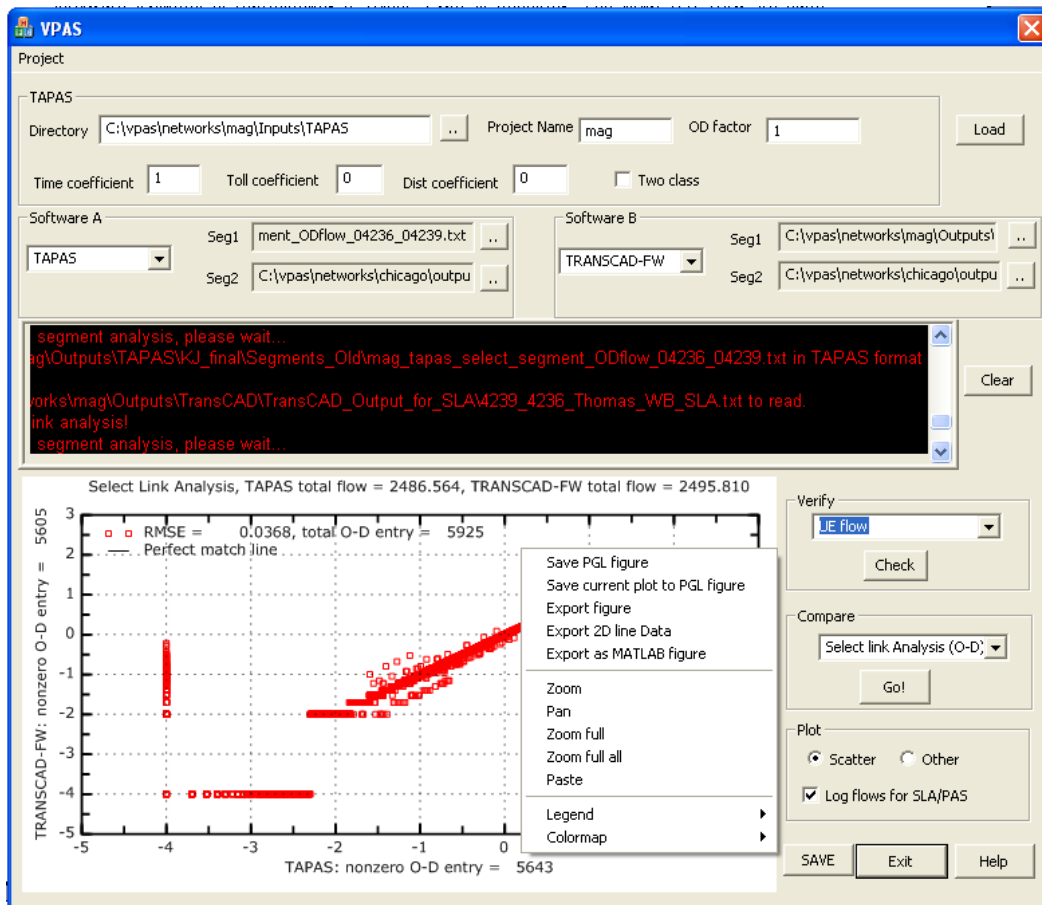


Figure 3.1 VPAS interface

3.2.3 Validation

If the two sets of user-equilibrium network assignment results from TAPAS and the practitioner software (CUBE, EMME, TransCAD, VISUM) are comparable, the link flows should agree. If the flows do not agree, differences may exist in the network file or OD file. To verify the assignment results from Software A and Software B, place the UE link volume files in the first segment for each software system (Seg 1 in Figure 3.1). Two types of format are supported: TAPAS format, which can be found in the total link flow file name_tapas_links.txt in the TAPAS output directory and a user-specified format assumed to be used by all other software systems. The file should have only one line of header and four columns:

TailNodeID HeadNodeID UELinkFlow UELinkCost

The above node IDs should be identical to those defined in the original software inputs, which may not agree with TAPAS inputs. VPAS can compare the link flow based on mappings provided by name_nid_SOFTWARE.dat.

The assignments can be verified by comparing link flows, link costs (given by the software), link costs calculated using link-delay function from given link volumes, and minimum OD costs for the link volume pattern. Objective functions, total travel time and absolute gap are also calculated when these comparisons are conducted. Choose from the “Verify” drop-down list and click “Check”.

Two types of plots, Scatter and Other, depending on which comparison is to be made, may be produced to visualize the results. In the Scatter plot for link volumes, for example, each point represents the vehicle flow per hour for a link as found by Software A and Software B; see Figure 3.2. The cumulative distribution function of the \log_{10} of absolute values of the differences of the selected item in the drop-down list (link flow, cost, or OD cost) between two software systems is shown if Other is checked; see Figure 3.3. This plot provides a more in-depth comparison of the two sets of link flows.

As shown in Figure 3.3, for about 20% of links the flows produced by the two software systems have a difference more than 1 vehicle per hour. Other statistics such as total travel times, objective function value (OFV), may also be found in these plots. The verification plots for link costs and O-D costs are similar.

When the plot appears, right-click on the plot to zoom in/out and export it into *.eps or *.png image format. A full name with extension must be given before exporting; e.g. Figure1.png.

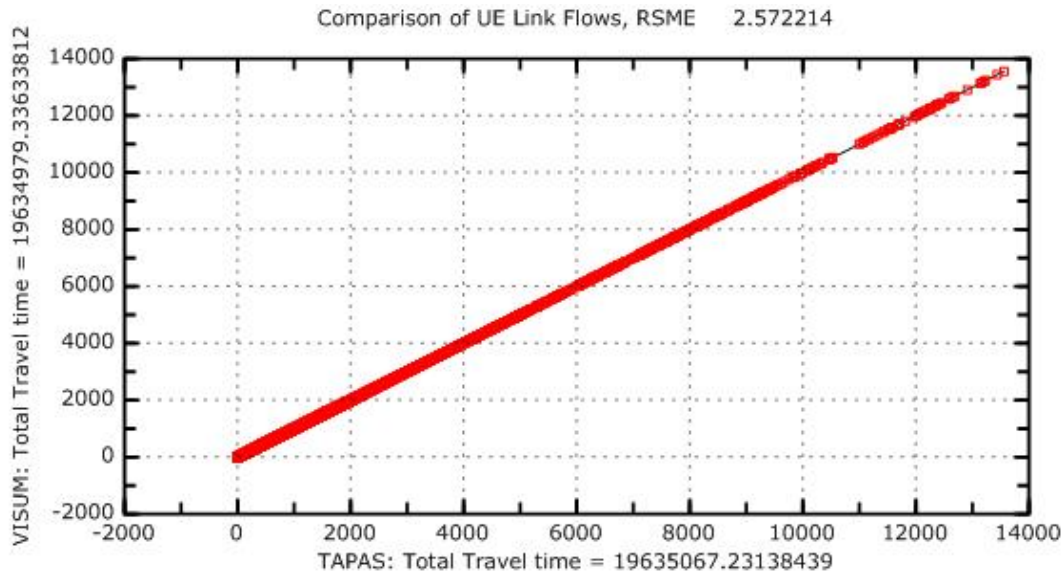


Figure 3.2 Scatter plot for link flow comparison

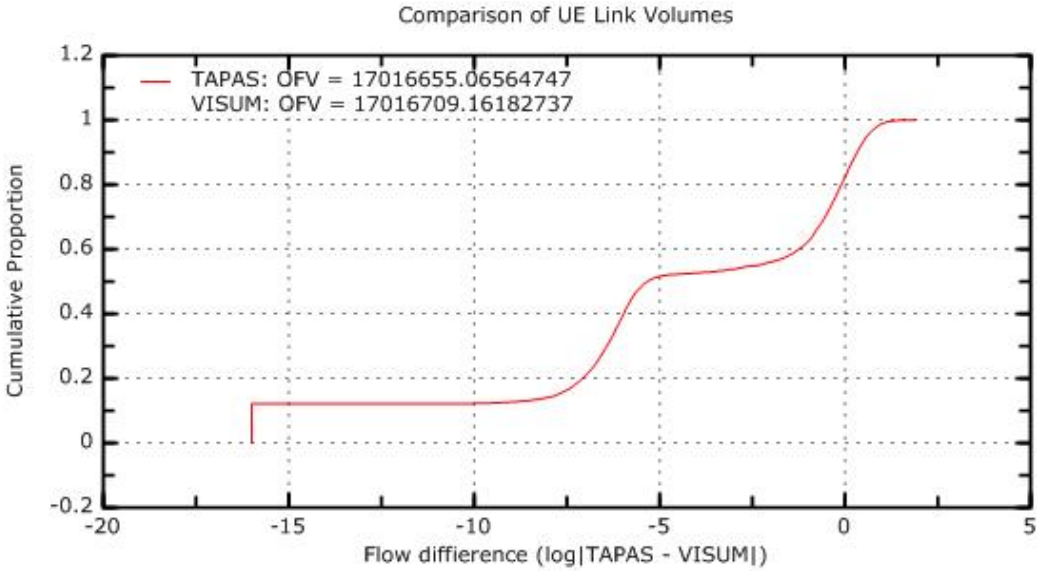


Figure 3.3 Cumulative log difference plot for link flow comparison

3.2.4 Select link analysis (SLA) and analysis of pairs of alternative segments (PAS)

Select link analysis (OD), pair of alternative segments analysis (PAS) and select link analysis (link) are used to compare SLA or PAS results from Software A and Software B. For select link analysis (OD or link), specify only one output file in Seg1 for each software. For PAS, specify the two output files at Seg1 and Seg2 for each software. That is, four files must be specified in total.

For SLA, each point in scatter plot represents the vehicle flow passing through the selected link for one OD pair as found by Software B (y-axis) and Software A (x-axis). The coordinates of each point show the vehicle flow for that OD pair on all routes that traverse the selected link. The numbers of OD pairs with at least one route traversing the selected link, as well as the total OD flow, as found by each software, are shown on the axis and title labels. The total number of OD pairs found by at least one software is shown in the body of the plot. Figure 3.4 gives an example. The scale of the plot can be either linear or \log_{10} . For \log_{10} scale, values less than $1E-4$, including 0.0, are shown as $1E-4$. Plots of this type are helpful in showing small OD flows. The use of the Other option produces a cumulative log difference plot as shown in Figure 3.5.

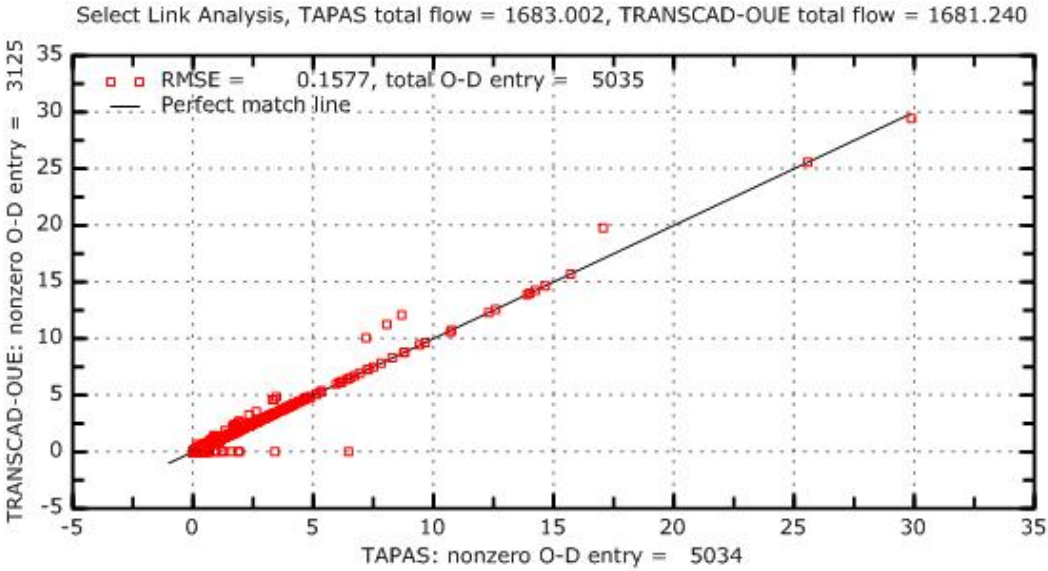


Figure 3.4 Scatter plot for select link analysis (OD) – linear scale

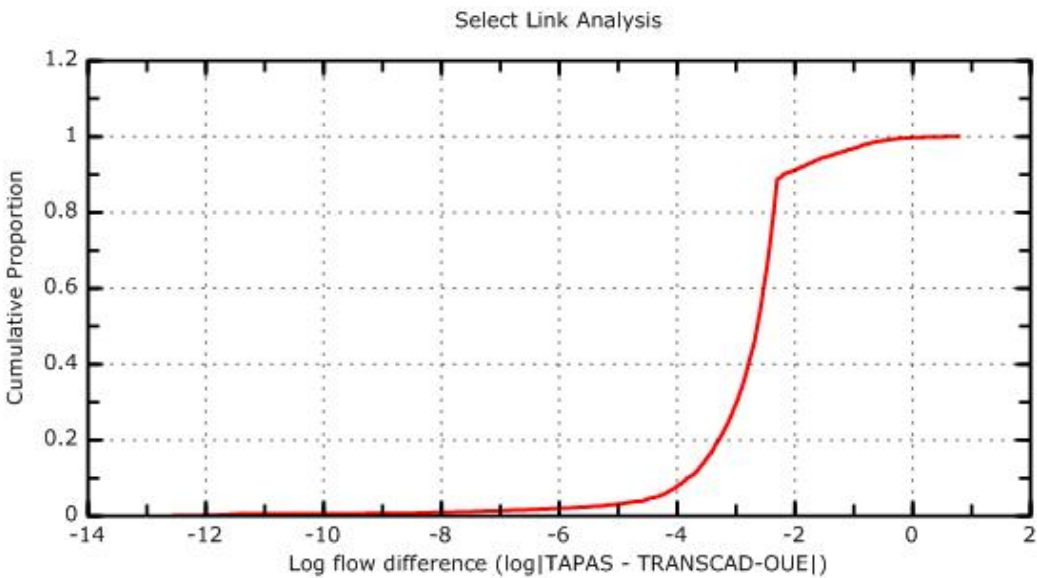


Figure 3.5 Cumulative log difference plot for select link analysis (OD)

For PAS analysis, each point in the scatter plot represents the vehicle flows for one OD pair passing through segment 1 (x-axis) and segment 2 (y-axis); see, for example, Figure 3.6. The two software systems are identified by color: red for Software A (default is TAPAS), and blue for Software B. The total flows passing through each of the two segments, as found by each of the two software systems, are given in the body of the plot. The Other option for PAS gives the log of the ratio of the OD flows that pass through each segment (y-axis) vs. the total OD flow (x-axis); see Figure 3.7. Note that: (1) an OD pair is included in the plot if and only if it is found to use either segment (or both) by either of the two software systems; (2) the ratio is bounded between 10^{-3} to 10^{+3} . Thus, a point on the horizontal lines $y = 3$ or $y = -3$ usually means the actual ratio violates the bound.

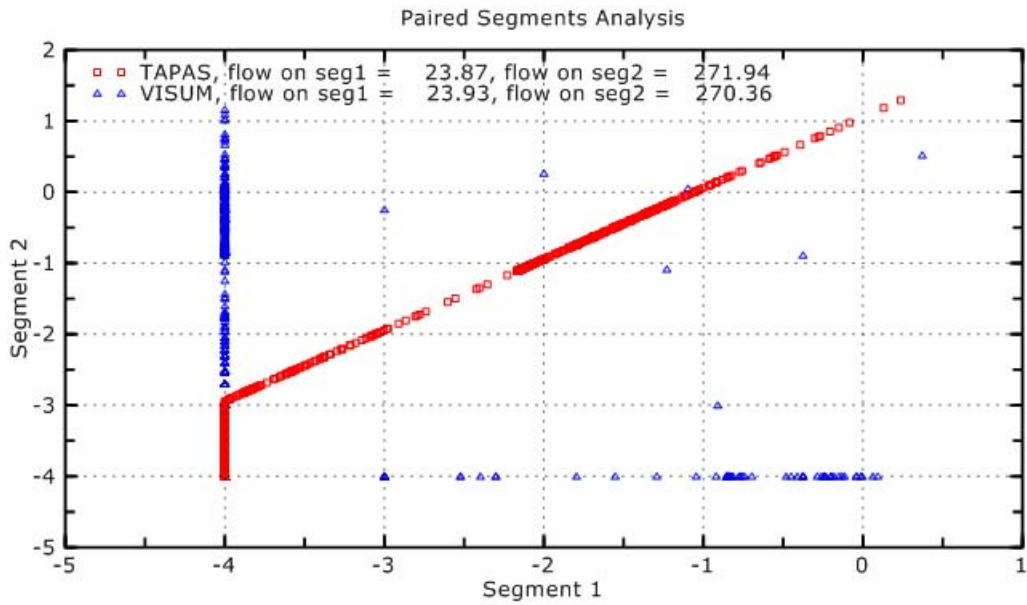


Figure 3.6 Scatter plot for a pair of alternative segments – log scale

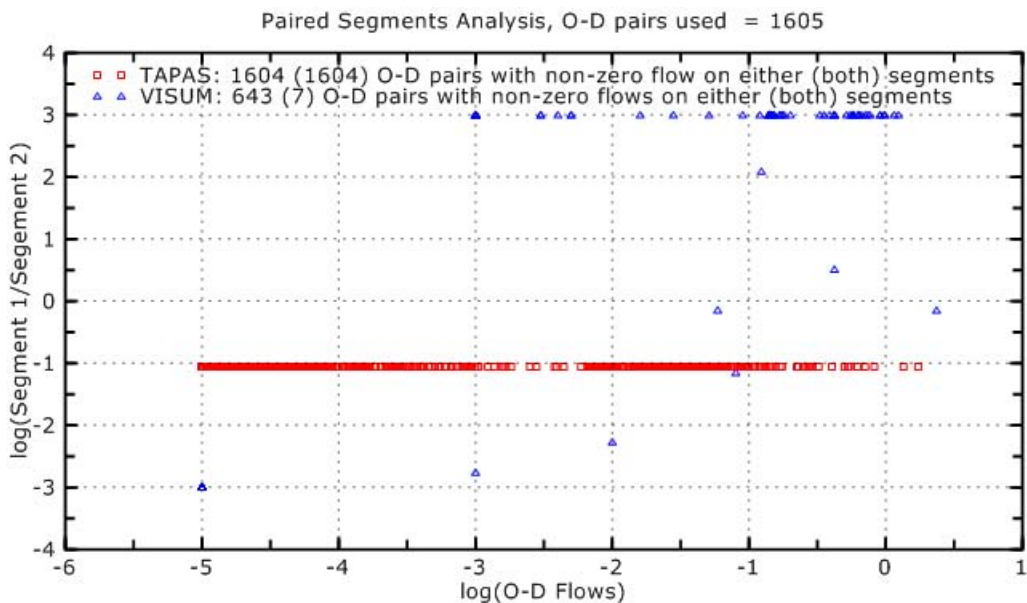


Figure 3.7 Log ratio plot for a pair of alternative segments

3.2.5 Multiple-class verification and SLA/PAS

Both TAPAS and VPAS provide limited support for multiple-class analysis. Presently, they can only assign two classes. To verify a two-class assignment in VPAS, check the “Two class” button before loading the project; see Figure 3.1. The Verification functions are valid in the two-class case only for link volumes and costs, not for other verification options. To conduct a select link analysis or a pair of alternative segment analysis for multiple-class assignment, the two class button should be UNCHECKED. These analyses proceed in the same way as for a single-class analysis, except the input flows, either for SLA or PAS, must be class-specific.

4. Collaborative Case Studies

4.1 Introduction

This chapter presents brief reports on each of the case studies undertaken by the six collaborators recruited by the investigators. The format and order of these case studies are described here.

The cases used to compare TAPAS with practitioner software systems may be broadly classified as one-class assignment and two-class assignment. Three case studies of each type were undertaken. In three of the six case studies, collaborators performed select link analyses to compare the OD-route flows found by TAPAS with those found by their practitioner software. These analyses are presented as the first results in these case studies. Then, four collaborators performed analyses of OD-route flows over a pair of alternative segments. These findings are presented next. In some of the studies, link flows corresponding to the OD-route flows are also presented. The sixth case study examined the effects of a more precisely converged assignment on user costs and benefits analysis of Build vs. No-build scenarios.

Among the six case studies, all four of the principal practitioner software systems in use in the United States were applied: CUBE, EMME, TransCAD and VISUM. Among the first three systems, only the assignment method generally known as the Frank-Wolfe or linear approximation assignment method was being used by the case study collaborators at that time. Accordingly, four of the case studies use this link-based method. Two studies applied the route-based method found in VISUM. In each case study, several analyses were performed. For the purposes of this report, the investigators selected a few analyses to display the principal findings. The case studies, collaborators, number of user classes, number of each type of analysis presented, software applied, and zone and network sizes are shown in the following table, which also shows the order of the presentation of results in the remainder of this chapter.

Table 4.1 Summary of the case studies

<u>Case Study</u>	<u>Collaborator</u>	<u>Class</u>	<u>SLA</u>	<u>PAS</u>	<u>Software</u>	<u>Zones</u>	<u>Links</u>
MAG	V. Livshits	1	2	1	TransCAD	2,006	39,015
PAG	A. Sun	1	2	2	TransCAD	870	10,584
DVRPC	C. Puchalsky	1	-	1	VISUM	2,068	20,245
PSRC-PTV	C. Joshi	2	-	2	VISUM	1,154	18,920
PSRC-RST	R. Tung	2	3	-	EMME	1,077	18,738
ODOT	R. Anderson	2	-	-	CUBE	395	21,250

Each case study begins with a brief model description, including the level of convergence of TAPAS and the practitioner software, the collaborator and objectives. The Relative Gap for TAPAS is stated in each study as the ratio of the Total Excess Cost (also known as the Gap) to the Total Cost; however, definitions of the Relative Gap vary among software systems. Then, the case study findings are presented. Finally, the conclusions of the collaborator are discussed. In each case, the collaborator submitted a report, ranging in length from 2 to 96 pages. From these reports, the examples presented here were selected, and the collaborators comments edited into a common format. In all case studies, the VPAS plots were redone from the files submitted by the collaborators, in order to assure uniformity in the formats and to verify the results.

4.2 Maricopa Association of Governments case study

4.2.1 Background, collaborator and study objective

The Maricopa Association of Governments (MAG) is the metropolitan planning organization for the metropolitan area centered on Phoenix, Arizona. The study was motivated by the importance of achieving comparability in forecasting traffic flows at the origin-destination-route level for planning and analytical tasks. Results of this evaluation will help MAG to make informed decisions on algorithmic choices in travel forecasting models.

This report presents the results of a limited case study comparing user equilibrium traffic assignments with fixed trip matrices prepared using the Frank-Wolfe (FW) assignment algorithm implemented in TransCAD 5.0 and a research code called TAPAS.

The 2007 MAG travel forecasting model was used for this case study. To simplify the testing environment, a typical BPR function with parameters, 0.15 and 4.0, was used in TAPAS and FW regardless of facility type and area type. The convergence targets and computing environments for this case study are shown in Table 4.2.

Table 4.2 Convergence targets and computing environment

Assignment Algorithm	TAPAS	Frank-Wolfe
Relative Gap	1.1E-9 ¹	2.5E-4 ²
Iterations	50	1000
Computing environment	Dell Precision T7400 Intel® CPU X5460 @3.16GHz, 3.25GB of RAM Microsoft Windows XP Professional Version 2002 Service Pack 3	

¹ Although the assignment was terminated at 50 iterations, additional iterations did not improve the convergence.

² The assignment was terminated at 1,000 iterations before the target Relative Gap of 1E-4 was achieved.

The number of zones in the MAG trip matrix is 2,006. A trip matrix for the morning peak period, 6:00 – 9:00 am, was used for the assignments; the total flow is 2,214,742.9 vehicles per period. The number of nodes in the road network is 12,005, and the number of links is 39,015. The case study was performed by Vladimir Livshits, Systems Analysis Program Manager and his staff.

4.2.2 Description of case study findings

A select link analysis compared the OD flows over East Thomas Road EB and WB.

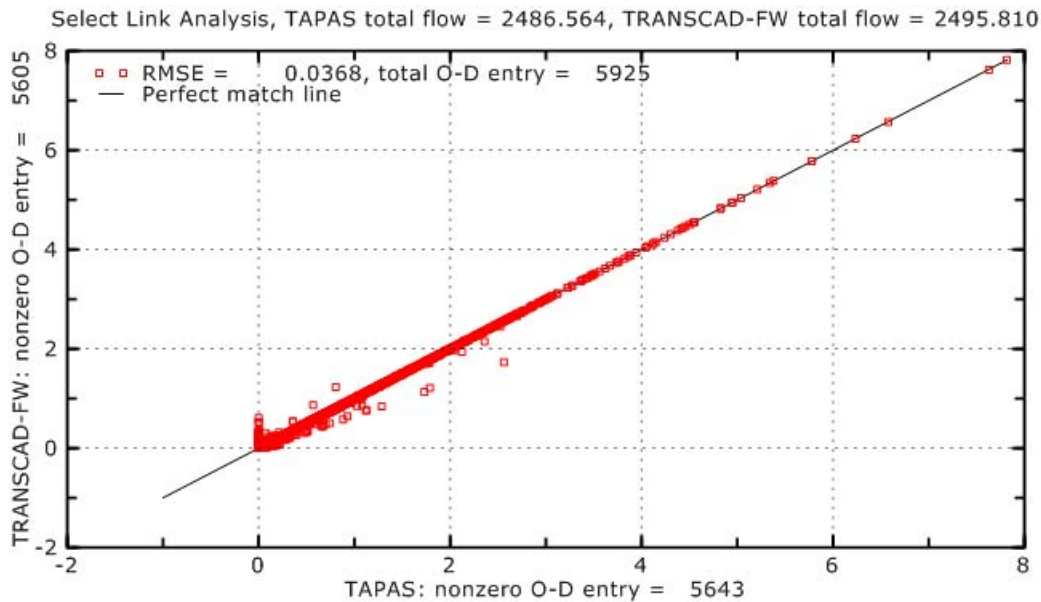


Figure 4.1a East Thomas Road eastbound – linear scale

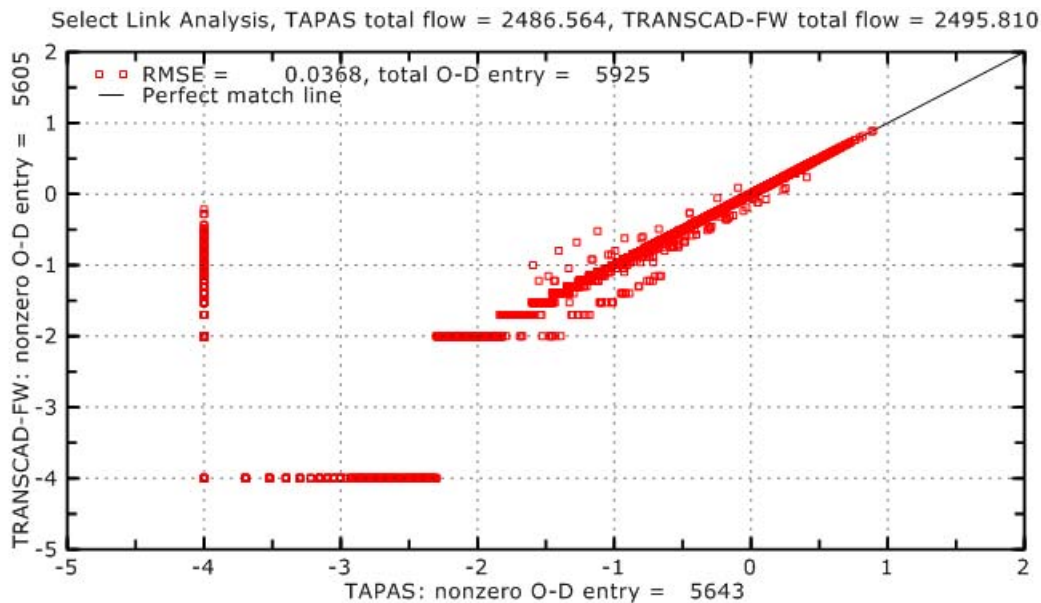


Figure 4.1b East Thomas Road eastbound – log scale

TransCAD found slightly fewer OD pairs than TAPAS, contrary to the general experience with the Frank-Wolfe algorithm. The total OD flows agree reasonably closely. As shown in the plots, the results agree very closely for flows over 2 vehicles per period. For smaller flows, a divergence occurs between the two results, perhaps in part because the OD flows were truncated to two decimal places. Since TAPAS is much more highly converged, one may infer that

TransCAD failed to find some OD flows, as shown by the horizontal line along the x-axis at $y = 1E-4$, and was unable to eliminate some residual flows shown by the vertical line at $x = 1E-4$.

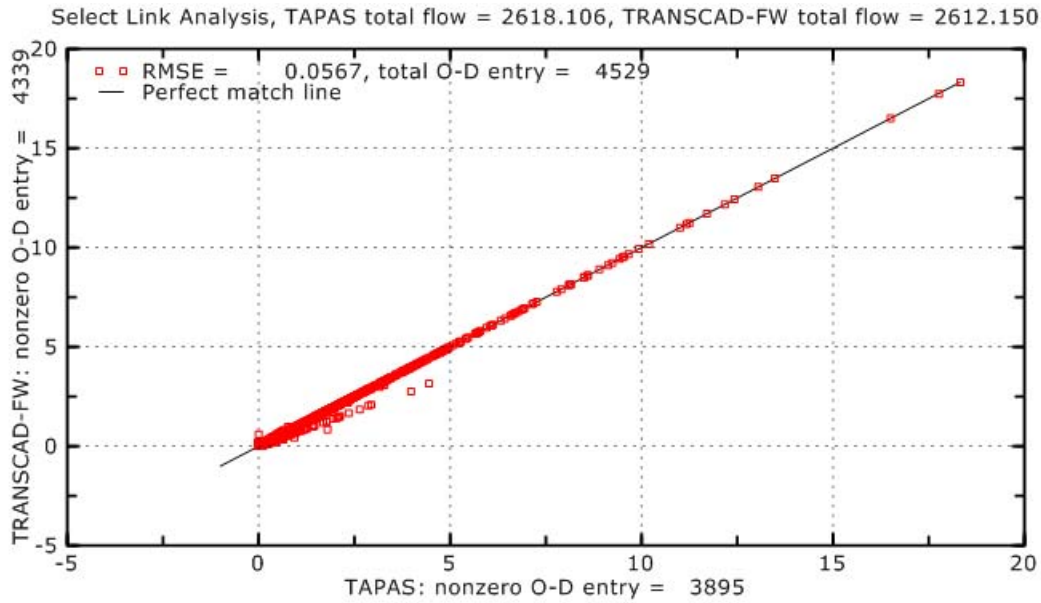


Figure 4.2a East Thomas Road westbound – linear scale

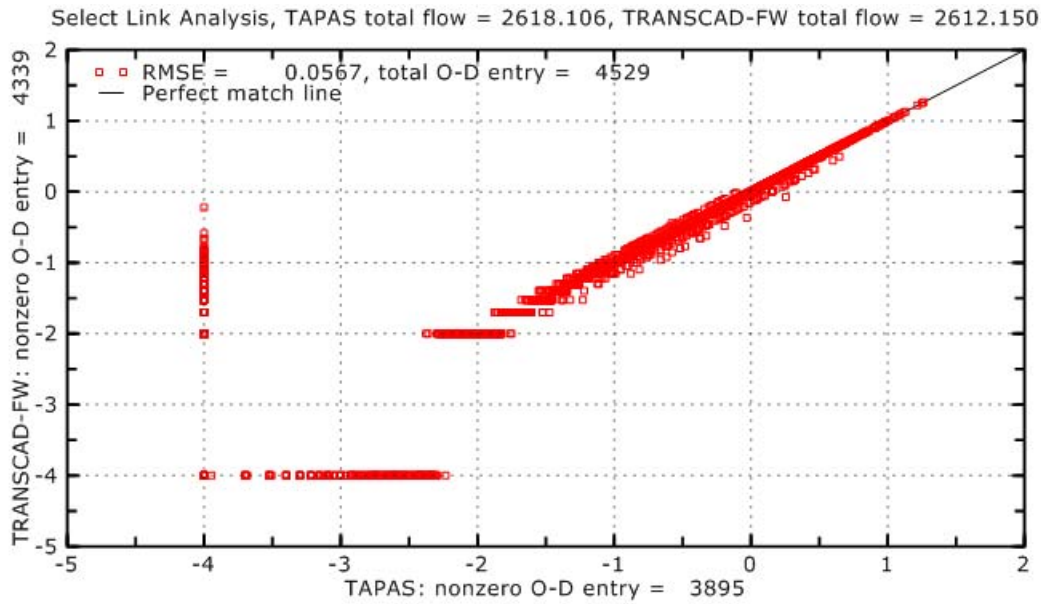


Figure 4.2b East Thomas Road westbound – log scale

The total OD flows agree reasonably closely, especially for OD flows greater than 5 vehicles per period. As for the EB case, the results diverge for OD flows less than 1 vehicle per period (shown as zero on the log scale).

An analysis of a pair of alternative segments was performed for the network shown in Figure 4.3.

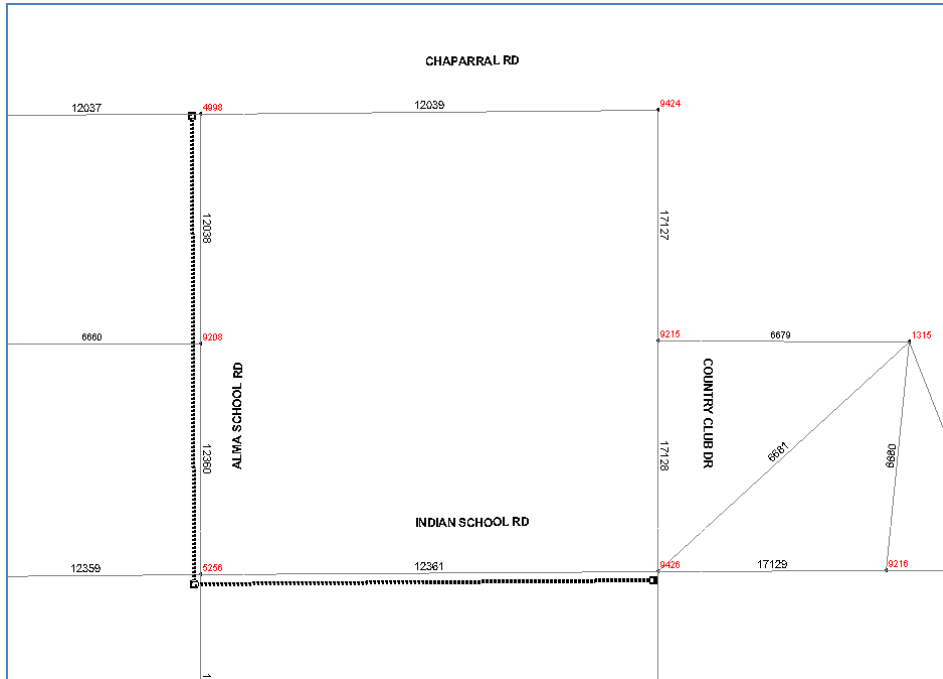


Figure 4.3a Segment 1 for PAS analysis

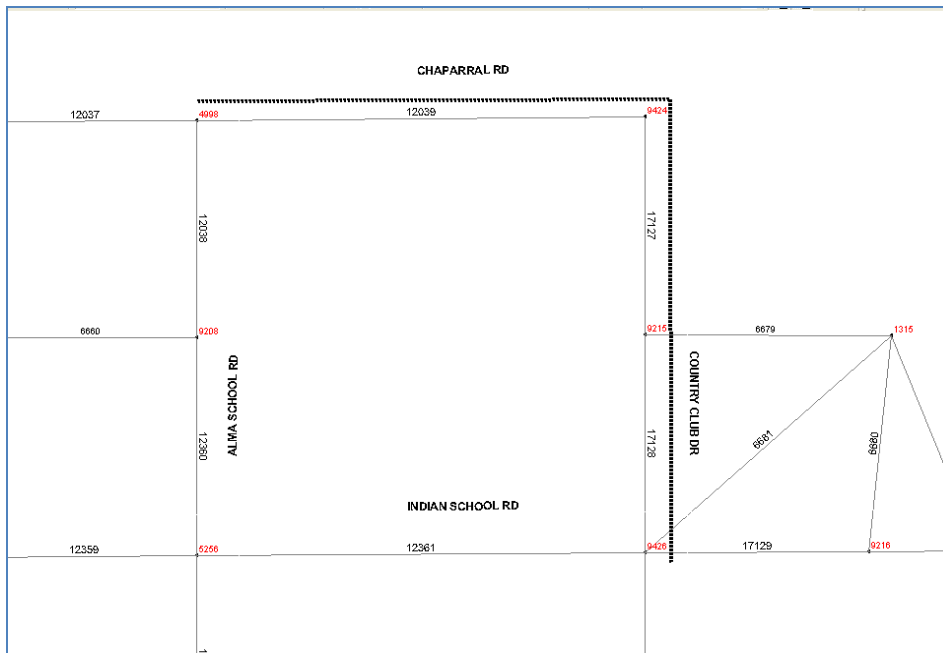


Figure 4.3b Segment 2 for PAS analysis

A comparison of the flows over the pair of alternative segments is shown in Figure 4.4.

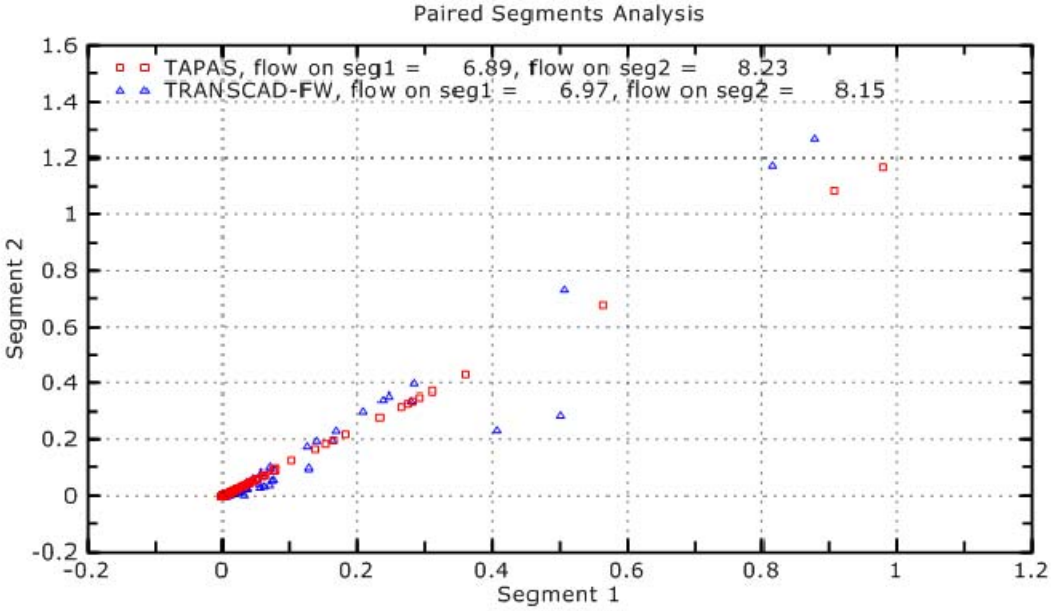


Figure 4.4a Flows over Segments 1 and 2 (vehicles per period) – linear scale

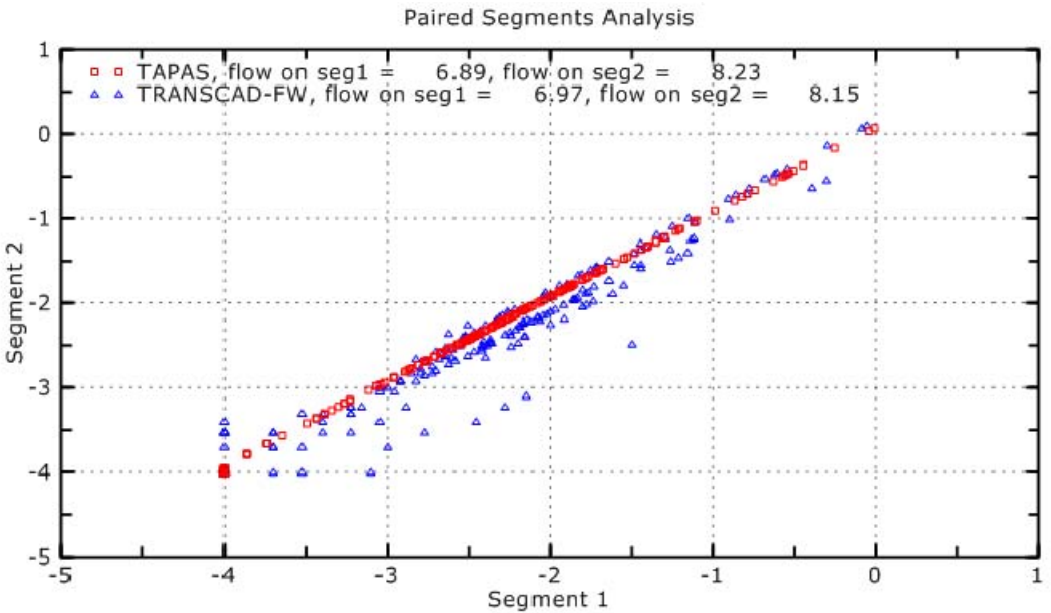


Figure 4.4b Flows over Segments 1 and 2 (vehicles per period) – log scale

TAPAS assigned the OD flows to the two segments in the same proportion, in contrast to somewhat different proportions found by the FW assignment. Figure 4.4c below, showing the log of the ratio of flows, illustrates the variation in the proportions more clearly. The horizontal line for TAPAS shows each segment flow has the same proportion. The scatter in the OD pairs for TransCAD illustrates the variations in proportions. Thirteen OD pairs have flow on only one segment.

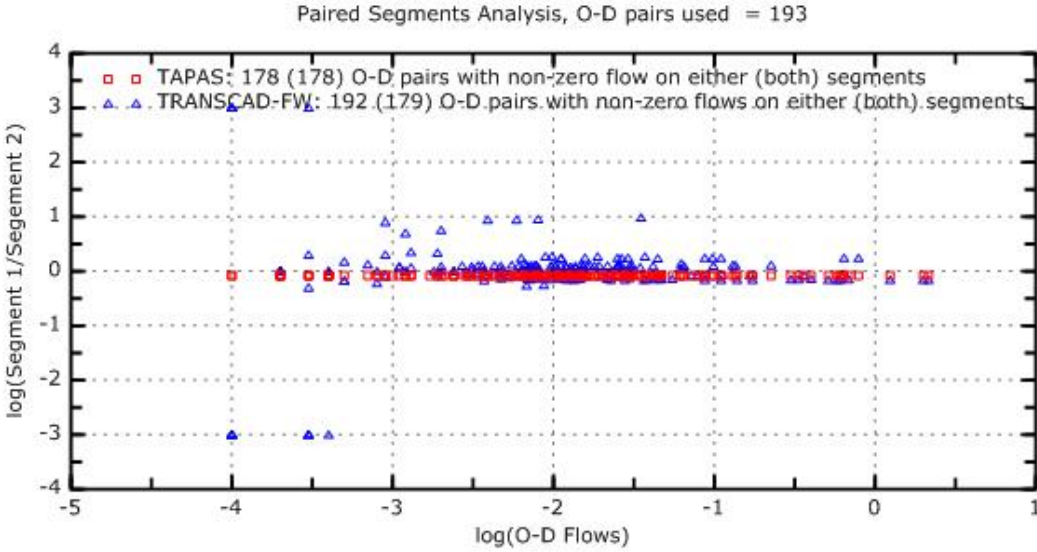


Figure 4.4c Log of the ratio of flows over segments 1 and 2

4.2.3 Conclusions of the collaborator

1. The select link comparison (Figures 4.1 and 4.2) between TAPAS and TransCAD-FW demonstrated satisfactorily close results, which may depend on the FW implementation; some deviations require further investigation.
2. The PAS analysis showed most of the segment flows found by TransCAD-FW (Figure 4.4) to be roughly proportional, as compared with the truly proportional flows found by TAPAS. More specifically, TAPAS assigned 178 OD pairs with positive flow to segments 1 and 2 with equal proportions; TransCAD-FW assigned 192 OD pairs to either or both segments 1 and 2, with 179 OD pairs assigned to both segments in varying proportions.
3. From MAG's perspective, decisions regarding practical applicability of the new algorithms and/or software require further investigation. The scope of this project did not cover practical issues affecting implementation of new assignment algorithms in regional travel forecasting models. However the results certainly warrant a more detailed, stringent and practical examination of the assignment algorithms, both in light of this project and several other relevant developments in commercial travel forecasting software.
4. TAPAS demonstrated a clear advantage in terms of computational effort required to reach precise convergence. TAPAS converged to a relative gap 1.1E-9, while TransCAD-FW reached a relative gap of 2.5E-4 at the maximum number of iterations specified (1000). TAPAS took 69% and 63% of FW's computation effort for SLA and PAS, respectively as shown in Table 4.3.

Table 4.3 Computational time (minutes)

Assignment Algorithm	TAPAS	TransCAD-FW
Select Link Analysis	40.9	58.9
Paired Alternative Segment Analysis	43.0	67.7

4.3 Pima Association of Governments case study

4.3.1 Background, collaborator and study objective

Pima Association of Governments (PAG) is the metropolitan planning organization for the metropolitan area centered on Tucson, Arizona. PAG maintains a regional travel forecasting model for 2008 for the evening peak hour, 5 to 6 pm, which was used for this study. The road network is shown as Figure 4.5.



Figure 4.5 Pima Association of Governments road network

The number of zones in the trip matrix is 870; the total flow is 258,734.3 vehicles per hour. Vehicle flows in the trip matrix are stated to two decimal places. The number of nodes in the road network is 3,644, and the number of links is 10,584. The case study was performed by Aichong Sun, Travel Forecasting Manager. TransCAD 5.0 was applied in assigning the trip matrices.

The case study examined vehicle flows over four selected links and three pairs of alternative segments (PAS), each comprised of sequences of four or more links. The objective of the analysis was to compare the vehicle flows over the selected links and the pairs of segments as determined by TransCAD and TAPAS.

The TransCAD link-based (Frank-Wolfe) assignment option was applied with a relative gap of $1E-5$; The TAPAS assignment was computed to a relative gap of $5.8E-17$. The TAPAS assignment results suggest the network is relatively uncongested for the assigned trip matrix.

4.3.2 Description of case study findings

Select link analyses

Select link analyses were performed for four arterial roads in the Tucson region: Aviation Parkway, Broadway, Grant Road and Oracle Road. Plots prepared with VPAS show the vehicle flows found by TAPAS and TransCAD 5.0 for two of these roads in Figures 4.6 and 4.7.

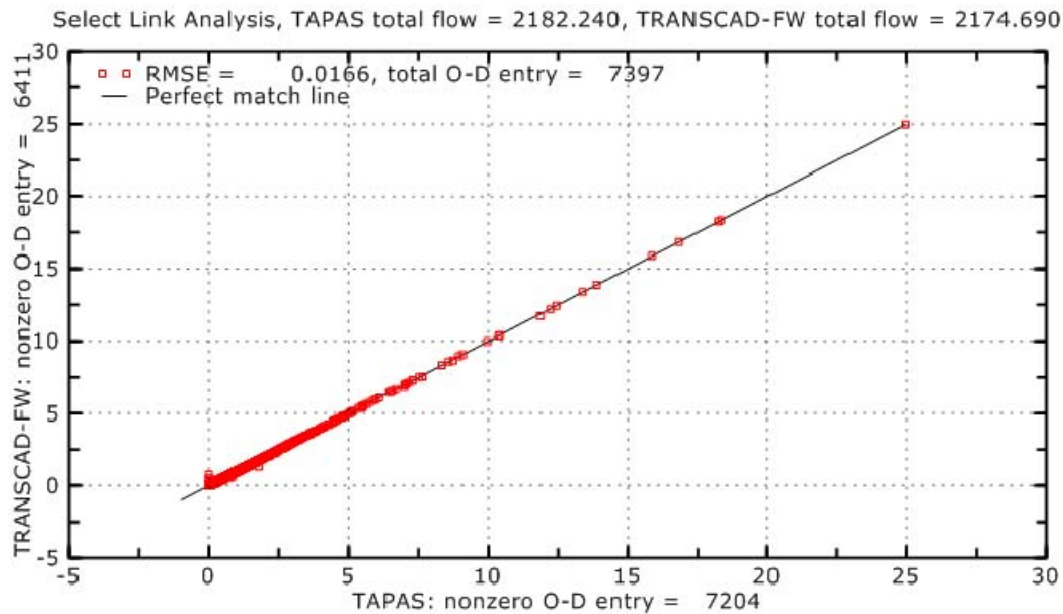


Figure 4.6a Broadway – linear scale

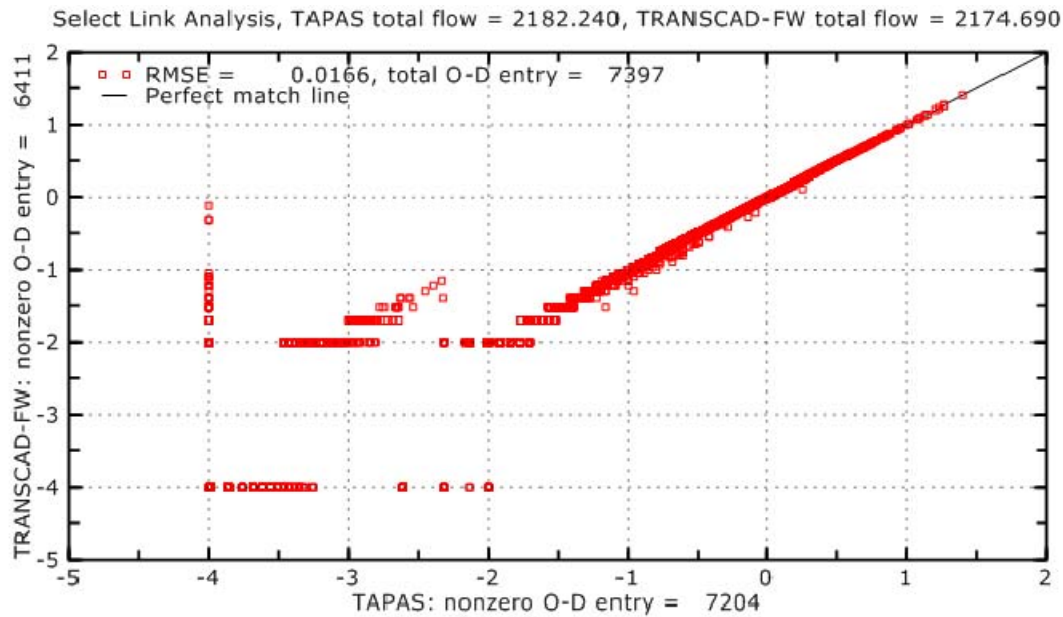


Figure 4.6b Broadway – log scale

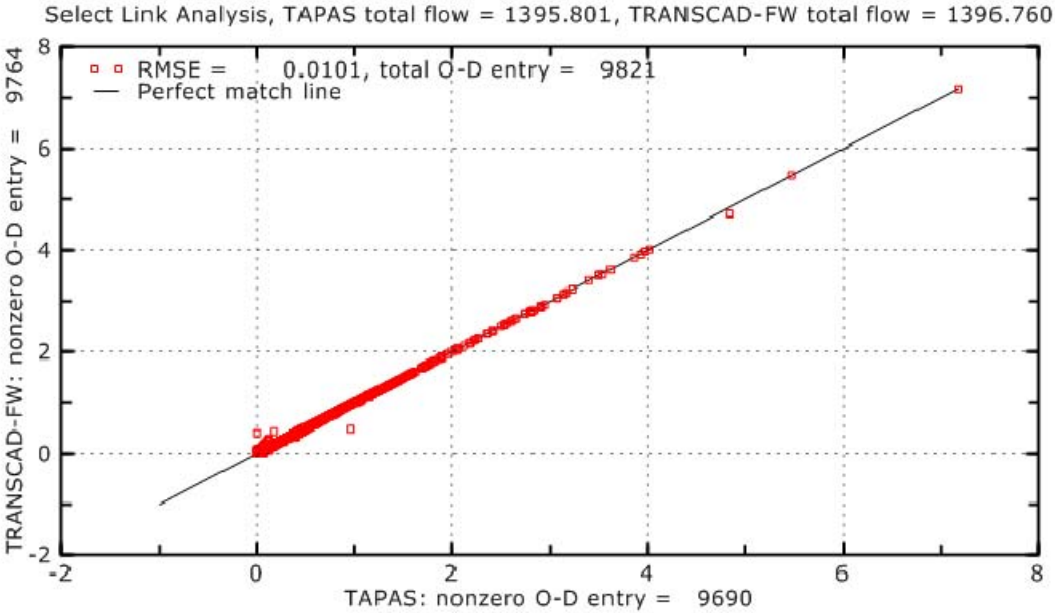


Figure 4.7a Grant Road – linear scale

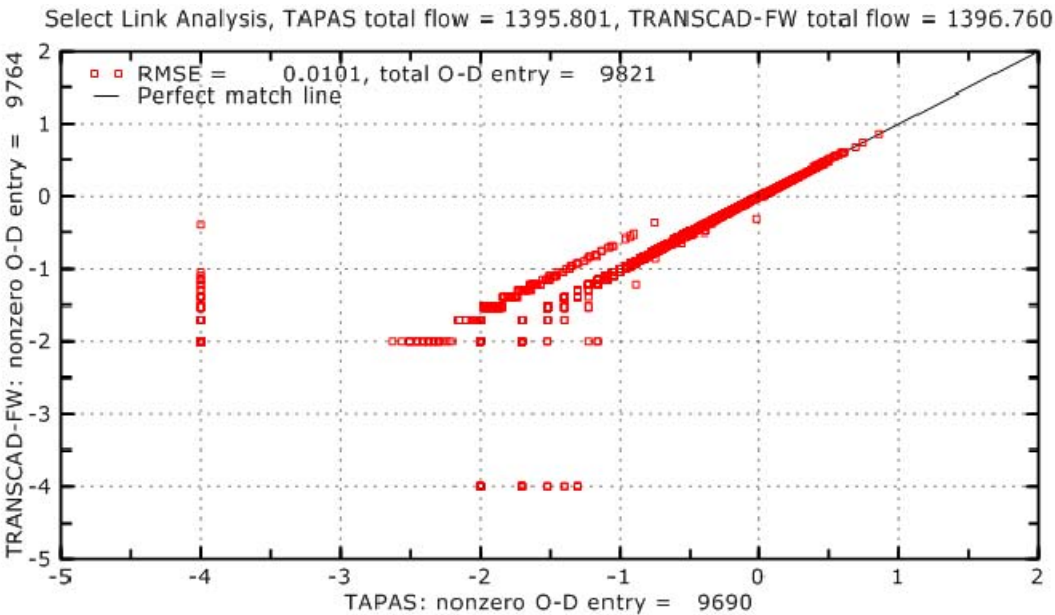


Figure 4.7b Grant Road – log scale

These two analyses show a close resemblance between the TransCAD assignment using the Frank-Wolfe method and the more highly converged TAPAS assignment in which the route flows are determined by the assumption of proportionality.

Vehicle flows greater than 1 vehicle per hour (zero on the log scale) are nearly identical in the two analyses shown. Flows less than 1 vehicle per hour vary somewhat. In addition, the TransCAD assignment finds some additional route flows in the Grant Road case, but fewer route flows in the Broadway case.

Pair of alternative segment analyses

1. Alvernon Way-Glenn Road vs. Grant Road-Dodge Blvd.

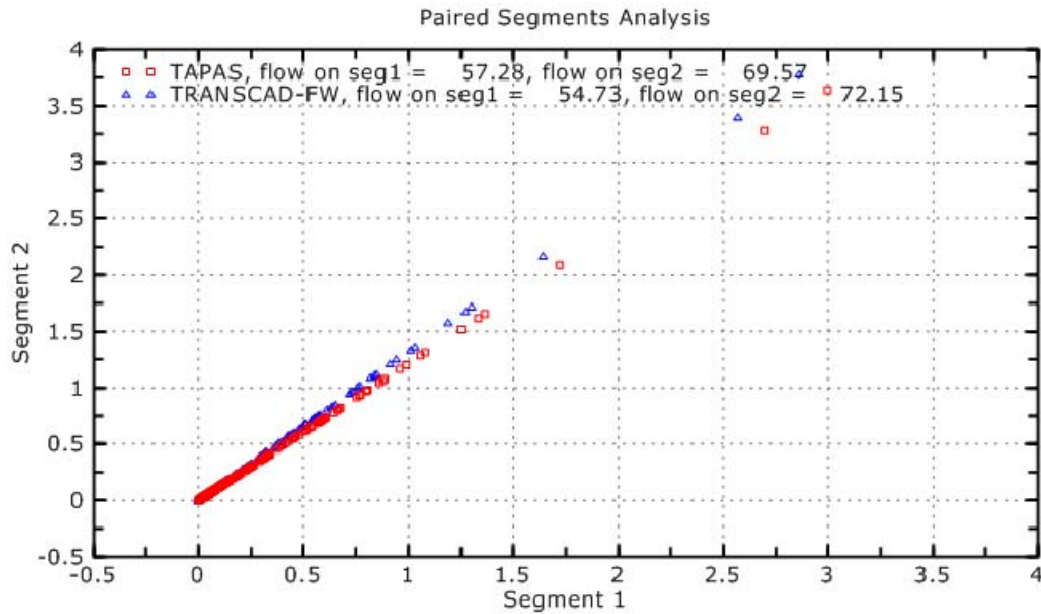


Figure 4.8a Alvernon Way-Glenn Rd. (seg. 1) vs. Grant Rd.-Dodge Blvd. (seg. 2) – linear scale

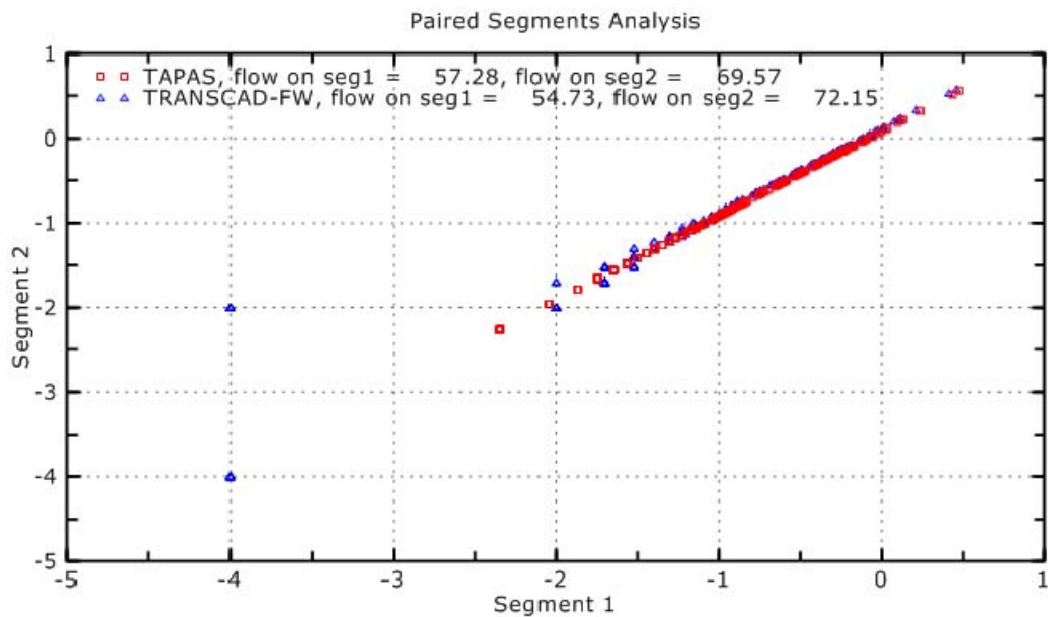


Figure 4.8b Alvernon Way-Glenn Rd. (seg. 1) vs. Grant Rd.-Dodge Blvd. (seg. 2) – log scale

The TAPAS solution is composed of 265 OD pairs; each segment has the same proportion of OD flows, as indicated by the alignment of the red squares in Figure 4.8b. The TransCAD-FW solution is composed of 257 OD pairs, of which 253 OD pairs have flows on both segments. As may be noted, some of the proportions of flows differ slightly among OD pairs; the reason is that in this application, the precision of the TransCAD output was limited to 1E-2.

The TransCAD values at $1E-4$ (log value = -4) correspond to cases in which there is no TransCAD OD flow on that segment. If there is a TransCAD OD flow on the other segment, or on TAPAS, then VPAS adds a zero flow to the TransCAD file. Even given these anomalies, the proportionality of the TransCAD solution is remarkable in that an assumption of proportionality was not a condition of the solution procedure.

2. Orange Grove Road vs. Ina/Skyline Road

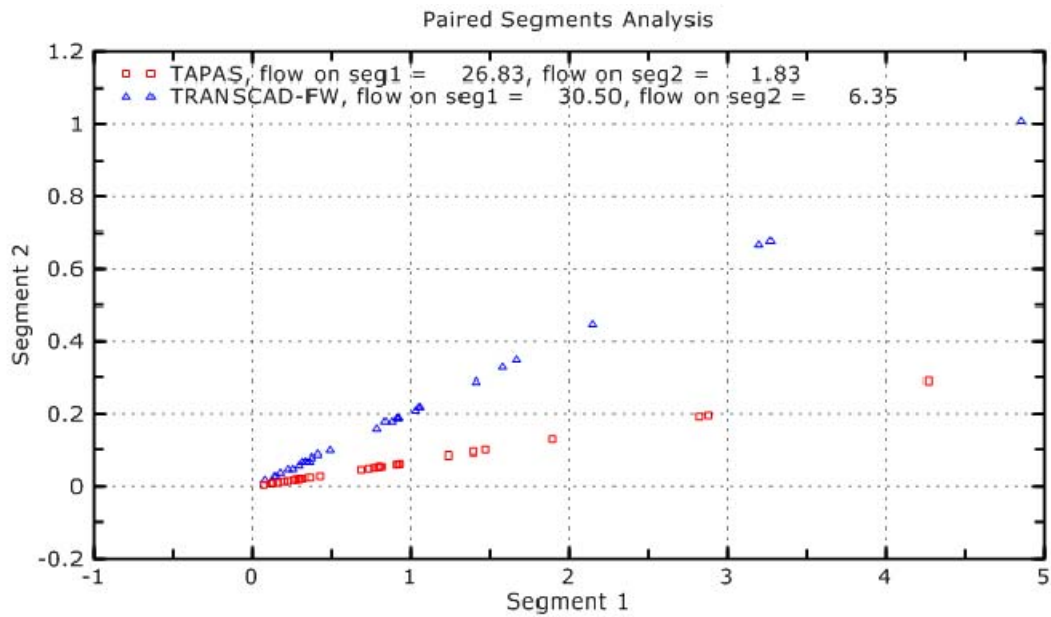


Figure 4.9a Orange Grove Road (segment 2) vs. Ina/Skyline Road (segment 1) – linear scale

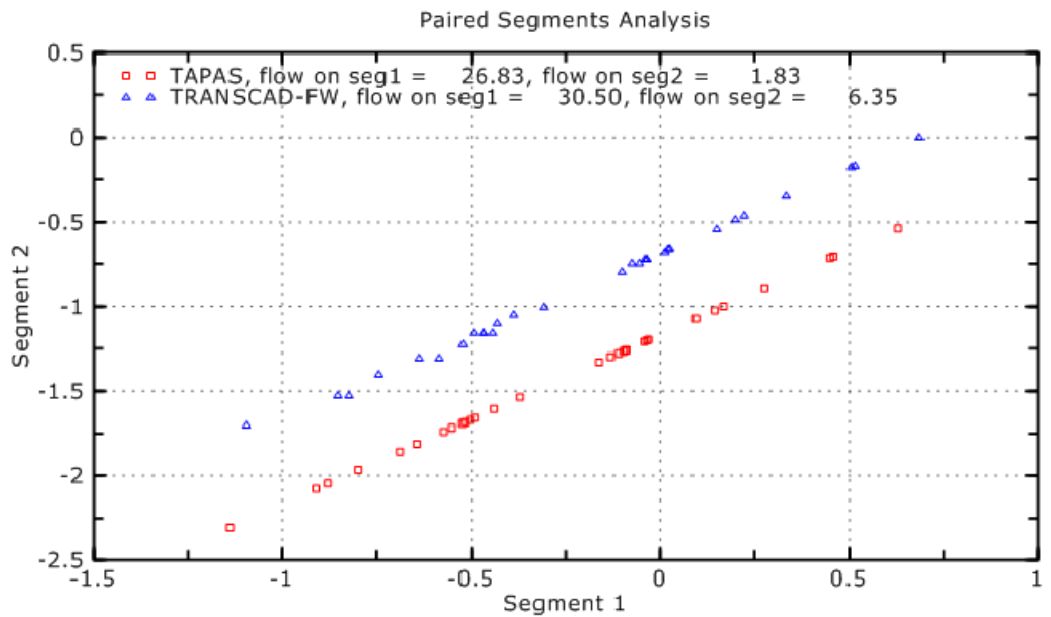


Figure 4.9b Orange Grove Road (segment 2) vs. Ina/Skyline Road (segment 1) – log scale

Both TAPAS and TransCAD-FW assigned flows from 30 OD pairs to this pair of alternative segments. For each assignment, the proportions of OD flow on the segments are highly similar. However, the proportion found for TAPAS is different from the proportion for TransCAD. Additional study is required to determine which solution better satisfies the assumption of proportionality.

In both of the PAS analyses presented here, the destination zone is the same for all origins. This fact, and the structure of the Frank-Wolfe algorithm, may explain why proportionality was achieved.

4.3.3 Conclusions of the collaborator

Both TransCAD-FW and TAPAS generated fairly similar results in terms of assigned link flow, which might be due to the fact that the network is not very congested. Select link analysis results from two assignment methods differ slightly, but the deviation is in reasonable range. Although the results do not provide evidence concerning which method is correct, the highly converged results of TAPAS avoid the residual flows observed in the select link analyses. In addition, the convergence of TAPAS is impressively faster than TransCAD-FW.

4.4 Delaware Valley Regional Planning Commission case study

4.4.1 Background, collaborator and study objective

Delaware Valley Regional Planning Commission (DVRPC) is the metropolitan planning organization for the southeastern Pennsylvania – southwestern New Jersey metropolitan area centered on Philadelphia. The DVRPC Regional Travel Demand Model was used for this study. At the time the case study was performed, this model was being converted from TRANPLAN to VISUM. The peak period, one-class highway trip matrix used with VISUM was estimated for 2005. This trip matrix and network were converted to the TAPAS input format. The generalized cost function used in the assignment utilized link travel time as determined by the BPR volume-delay function; in addition, Delaware River bridge tolls were converted to equivalent delays in minutes. The number of zones in the trip matrix is 2,068; the total flow is 7,338,247 vehicles per period, where the period is a five-hour period composed of travel during 7-9 am and 3-6 pm. The number of nodes in the road network is 20,245, and the number of links is 54,405. The case study was performed by Christopher Puchalsky, Manager, Modeling and Analysis.

The case study examined vehicle flows over a pair of alternative segments (PAS), each with sequences of four links, and terminating at a zone in West Philadelphia near the University of Pennsylvania. The PAS was determined to have the same travel time over each segment in a highly converged solution with TAPAS. The objective of the analysis was to compare the vehicle flows over the pair of segments as determined by VISUM 10.0 and TAPAS.

The VISUM route-based assignment option was applied with a maximum relative gap of 1E-3, a maximum of 20 iterations and four balancing iterations; subsequently, the maximum relative gap was reduced to 1E-4 with a maximum of 40 iterations and 10 balancing iterations. Results are presented for both solutions to show the effect of the more precisely converged solution. VISUM's flow bundling procedure was used to extract the OD flows for the pair of alternative segments. The relative gap of the TAPAS solution is 8.0E-12.

4.4.2 Description of case study findings

The location of the case study is shown in Figure 4.10; Zone 143 is shown to the left on the map.

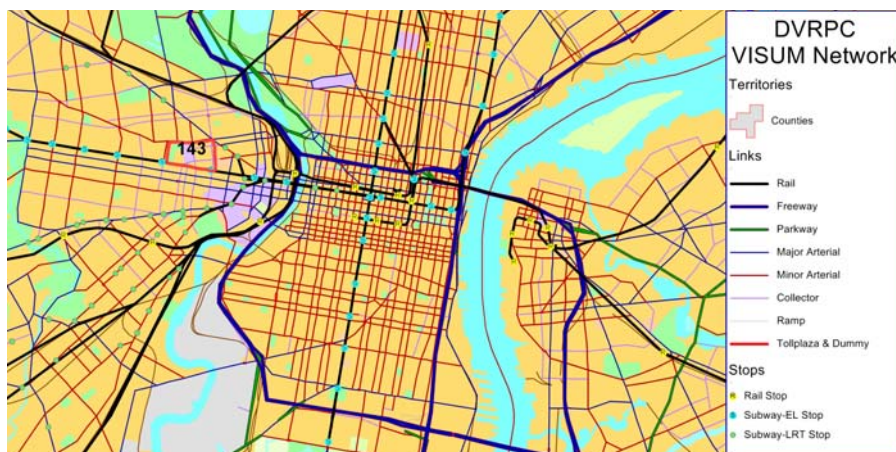


Figure 4.10 Location of the case study in central Philadelphia

The map of the pair of alternative segments is shown in Figure 4.11. The links are depicted as red lines, except for the two centroid connectors pointing into zone 143. The location of the initial and terminal nodes of the PAS are shown by ovals on the right and left sides of the map, respectively.

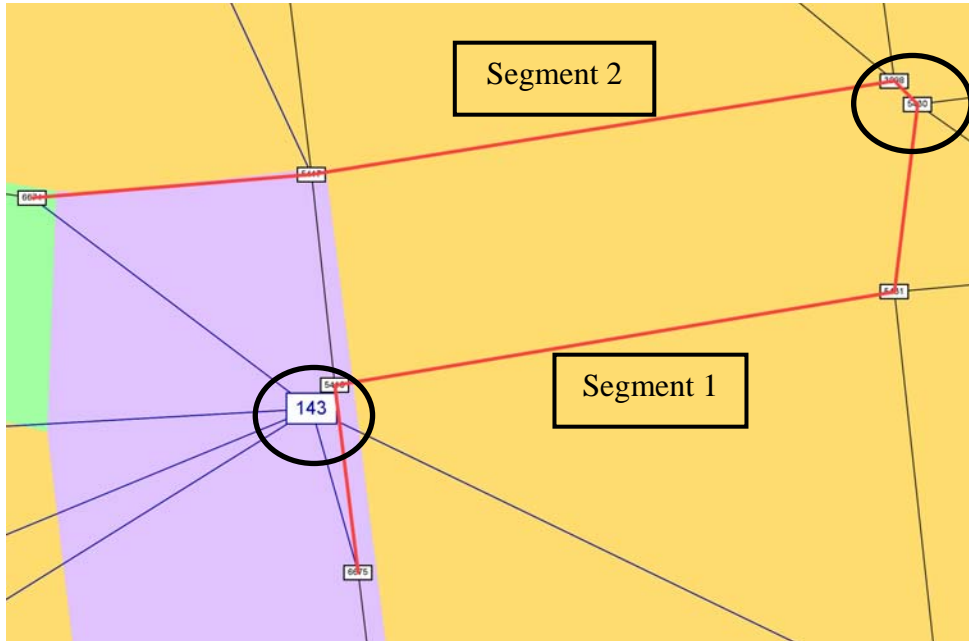


Figure 4.11 Pair of alternative segments connecting node 5430 to zone 143

Using the flow bundling procedure, the flows leading into node 5430 were identified for segments 1 and 2. They are shown in Figure 4.12. If the flows on each segment were proportional for each OD pair, then the links with flow in Figure 4.12 would be identical. Clearly, this is not the case.

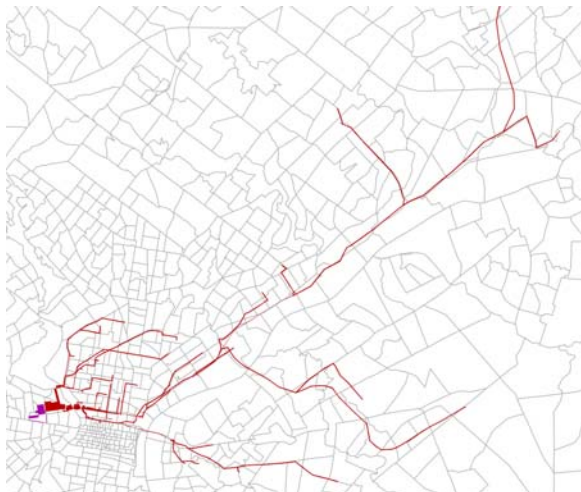


Figure 4.12a Flow bundle for segment 1

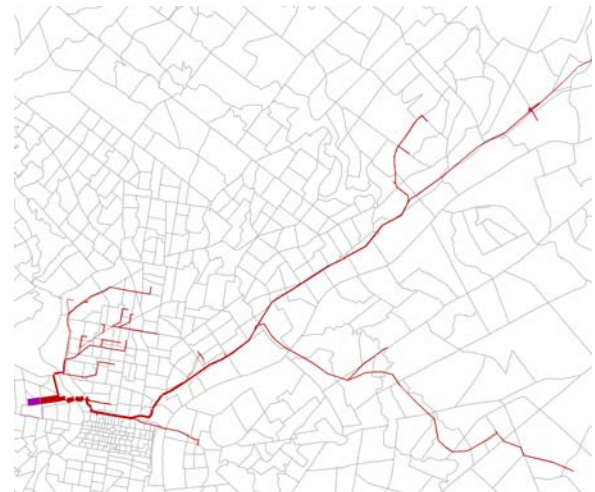


Figure 4.12b Flow bundle for segment 2

Following the extraction of the OD flows using each segment in VISUM and TAPAS, the flows on each segment were plotted with VPAS, as shown in Figure 4.13. The segment flows given by TAPAS are proportional for each of the eight OD pairs shown. For the VISUM flows for the solution with Relative Gap of 0.001, each OD flow occurs on one segment or the other, which might be termed an all-or-nothing assignment of route flows. Moreover, the total flow determined by the two algorithms is quite different: 185.0 for TAPAS vs. 158.0 for VISUM, although the flows on segment 1 are similar.

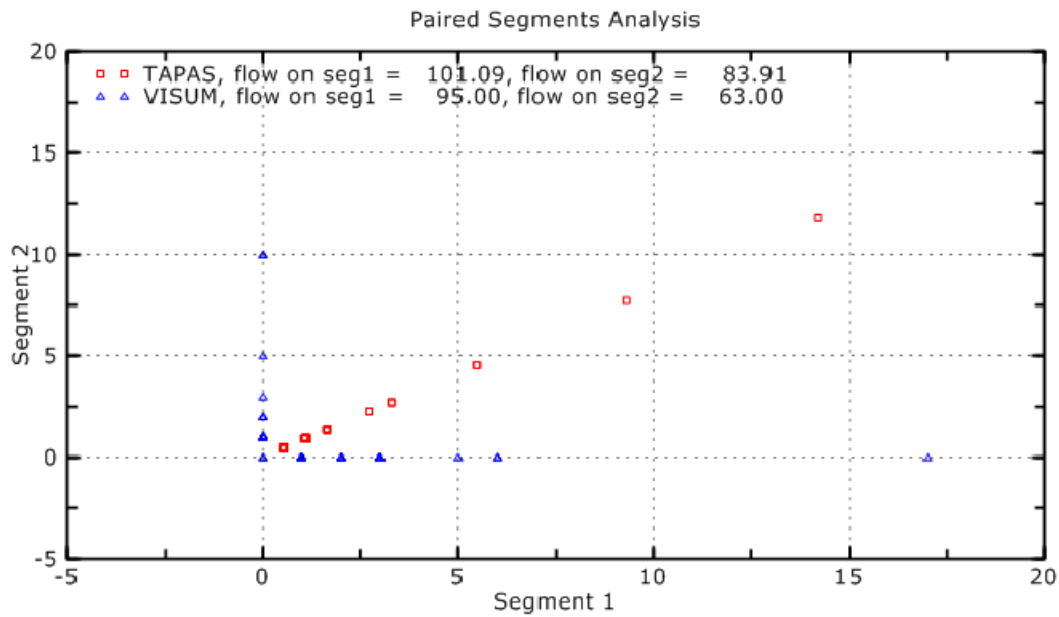


Figure 4.13 OD flows on segments 1 and 2 for TAPAS and VISUM (1E-3) – linear scale

The results for VISUM with a Relative Gap of 1E-4 are shown below in Figure 4.14.

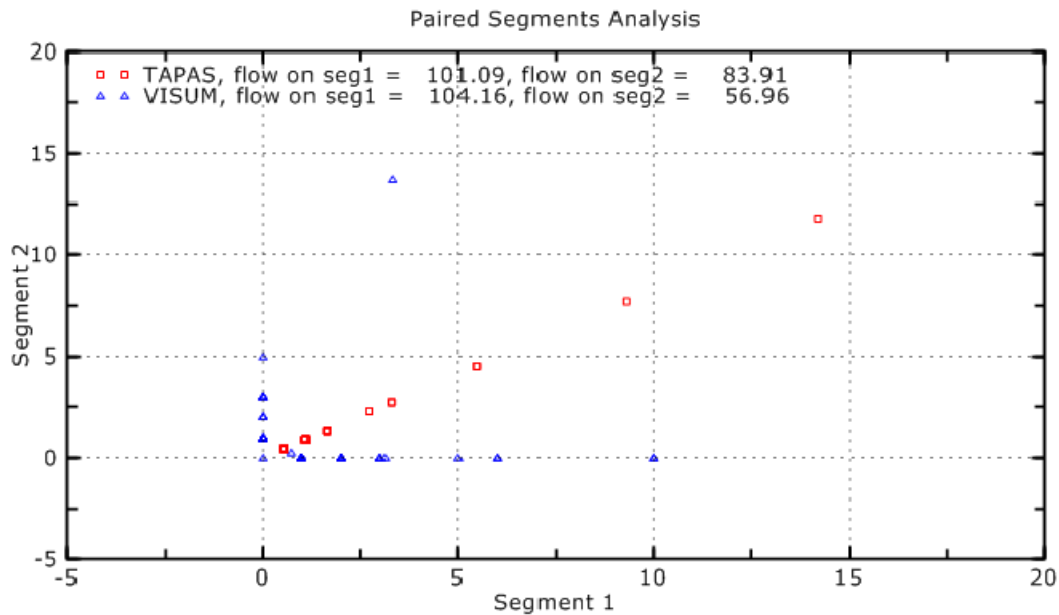


Figure 4.14 OD flows on segments 1 and 2 for TAPAS and VISUM (1E-4) – linear scale

The results for VISUM shown in Figure 4.14 are rather similar to those in Figure 4.13, except that one OD pair now has route flows on both segments. The total flow in the VISUM solution increased slightly to 161.1 from 158.0. Note that the VISUM flow on segment 1 now slightly exceeds the flows found by TAPAS.

Another way of displaying these results is in terms of the logarithm of the segment flows, as shown in Figures 4.15 and 4.16 below.

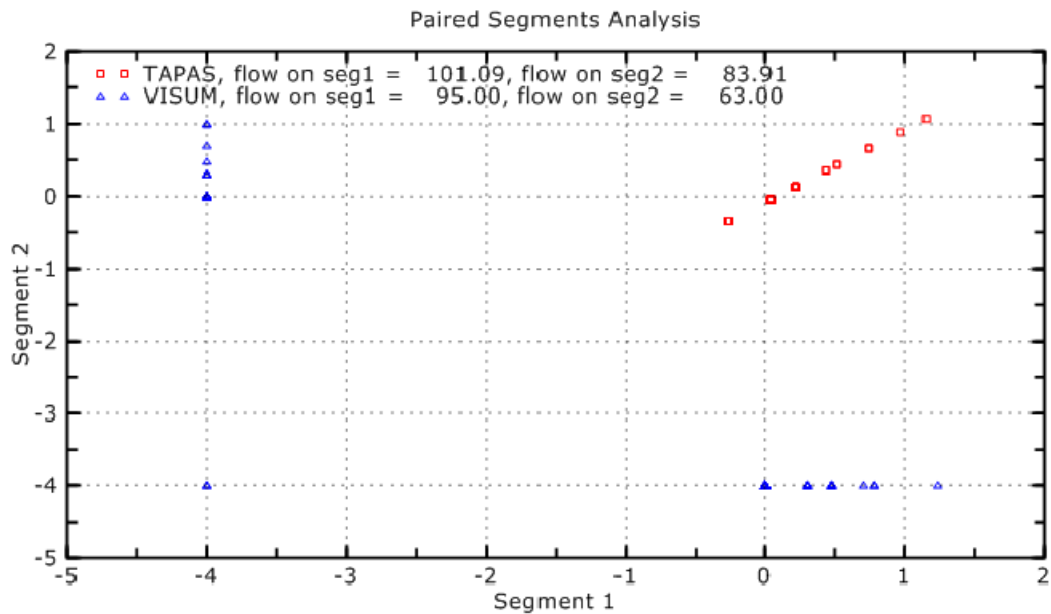


Figure 4.15 OD flows on segments 1 and 2 for TAPAS and VISUM (1E-3) – log scale

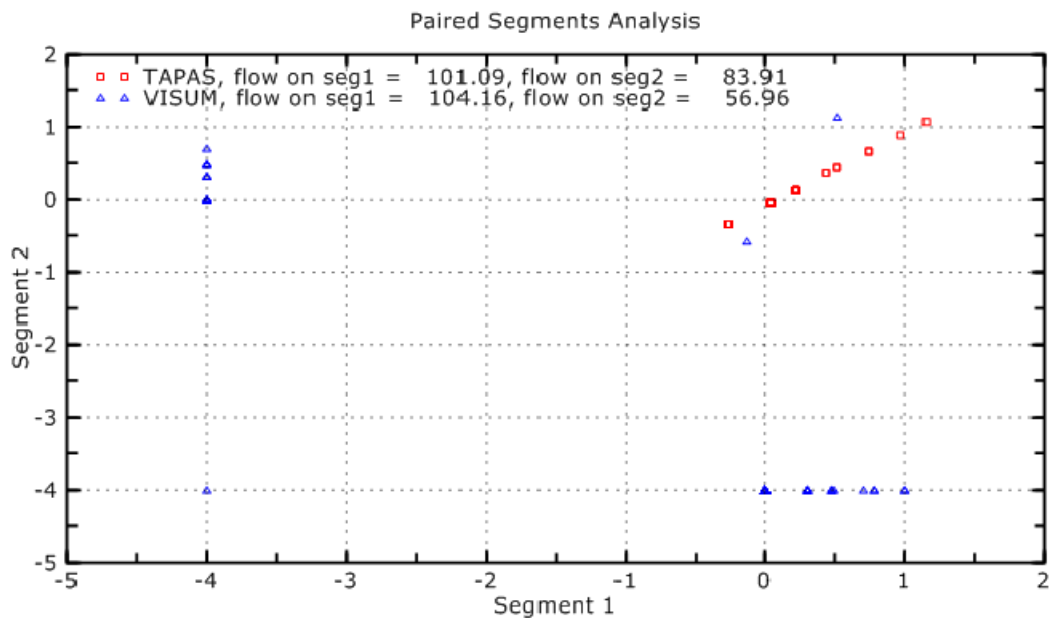


Figure 4.16 OD flows on segments 1 and 2 for TAPAS and VISUM (1E-4) – log scale

4.4.3 Conclusions of the collaborator

As was shown above, TAPAS and VISUM distribute flow among alternative segments of a PAS quite differently: VISUM tends to assign the flow in an all-or-nothing fashion for the various O-D pairs that use the PAS, whereas TAPAS tends to split each O-D pair's flow proportionately between the alternative segments.

Select link analysis is a significant part of the analysis that performed by DVRPC for project level work. If TAPAS is taken as the "correct" solution, these results indicate that our current analysis may be leaving out part of the picture. It will be interesting to see what will happen once DVRPC changes from the current route-based assignment algorithm in VISUM to the new LUCE algorithm.

Also as noted above, the total flow using the PAS differs between the two results. Since the difference is not caused by different trip matrices, this difference seems to indicate different flow patterns between the software systems on a more macro basis than just the PAS level. This result, of course, pertains only to one small example. More studies are needed to confirm these results.

4.5 Puget Sound Regional Council – PTV America case study

4.5.1 Background, collaborator and study objective

The Puget Sound Regional Council (PSRC) is the metropolitan planning organization for the Seattle-Tacoma metropolitan area in the State of Washington. PTV America, one of the four principal providers of travel forecasting software to US planning agencies, consultants and universities, recently converted the PSRC 2006 model to its VISUM platform. Robert Shull, Vice-President of PTV America at its Tacoma office offered to use this model in a collaborative case study to compare the VISUM results with results from TAPAS. Chetan Joshi, VISUM Product Manager, converted the VISUM model into TAPAS inputs, and obtained the OD flows and total link flows with TAPAS and VISUM for several cases. His findings and interpretation are presented below.

The version of the PSRC 2006 model implemented in VISUM 10.0 has 1,154 zones, 6,248 nodes and 18,920 links. The total single occupancy vehicle (SOV) flow is 1,330,620.89 vehicles per three hour peak period, which represented the AM peak period. The total high occupancy vehicle (HOV) flow is 208,706.46 vehicles per three hour peak period. The TAPAS implementation has the same number of zones, nodes and links. In the TAPAS network representation, 668 HOV links were blocked from SOV flows.

The VISUM route-based option was used in this collaboration. The assignment reported a Relative Gap of 2.9E-5 for SOV and 7.0E-6 for HOV. TAPAS solved the assignment with a Relative Gap of 1.1E-10.

Analyses of pairs of alternative segments (PAS) were performed for several subnetworks using VISUM and TAPAS. Two of these are presented below.

4.5.2 Description of case study findings

Case 1. The pair of alternative segments is shown in Figure 4.17.

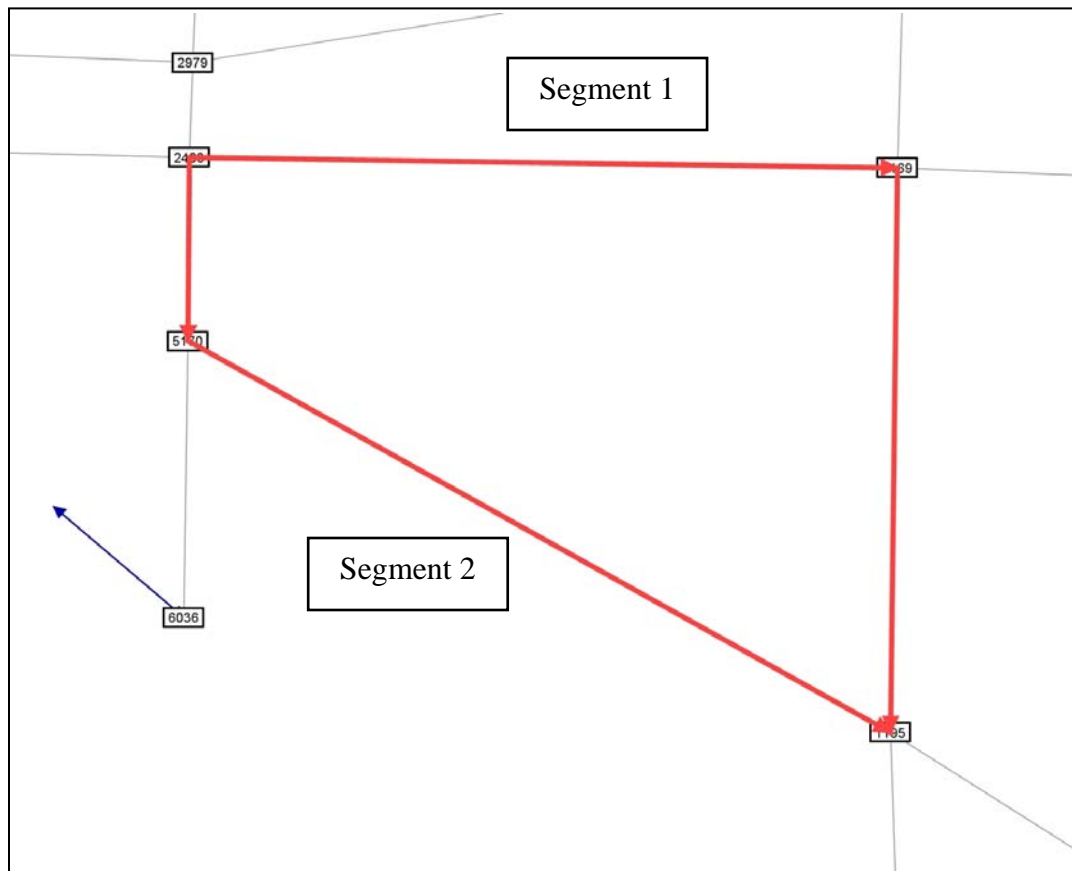


Figure 4.17 Segments 1 and 2 linking node 2403 to node 1195

Four sets of figures show the SOV and HOV route flows for this case:

1. SOV and HOV OD flows for segments 1 and 2 at linear and log scales (Figures 4.18 and 4.19);
2. total link flows using segments 1 and 2 at linear and log scales (Figures 4.20 and 4.21).

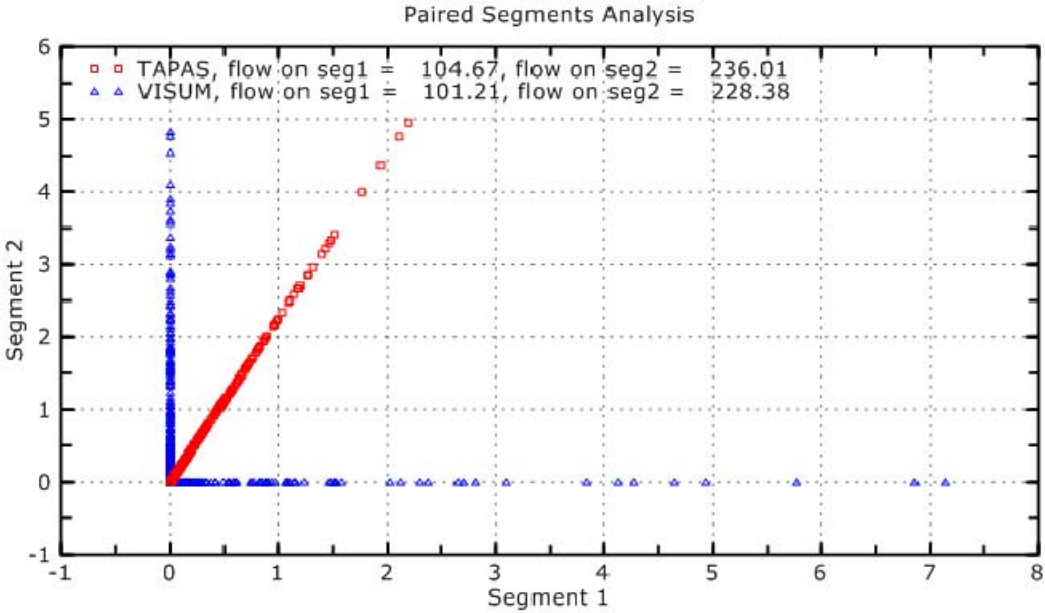


Figure 4.18a SOV OD flows on segments 1 and 2 by TAPAS and VISUM – linear scale

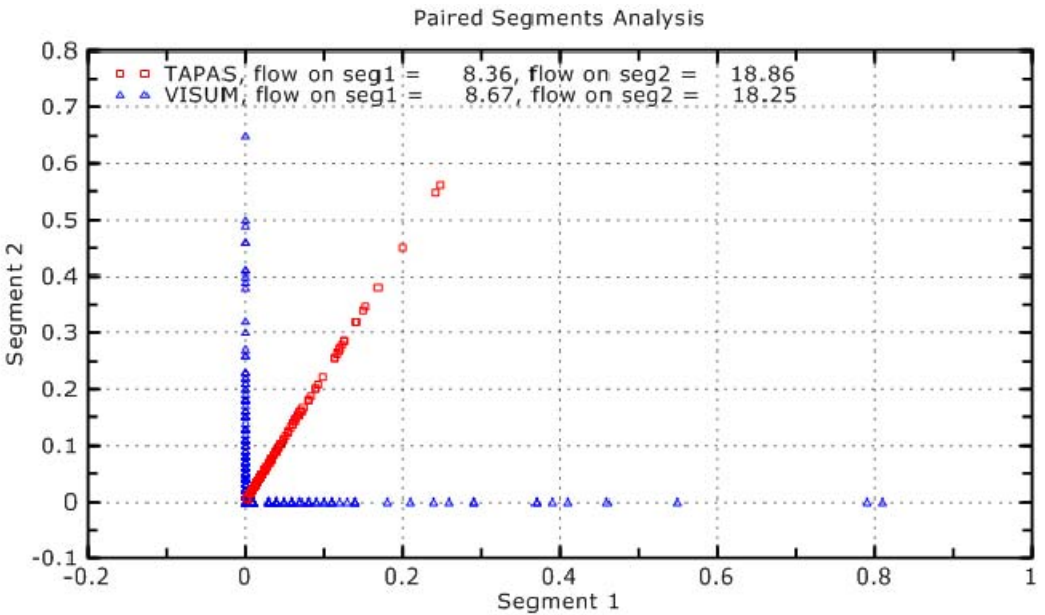


Figure 4.18b HOV OD flows on segments 1 and 2 for TAPAS and VISUM – linear scale

Note that in this case for both SOV and HOV flows, VISUM places each OD flow, represented by triangles, on one segment or the other. In contrast, TAPAS places the same proportion of each OD flow on a given segment for each OD pair, as shown by the squares. The total OD flows on each segment found by each algorithm is effectively equal.

Table 4.4 Ratio of total segment flows for VISUM and TAPAS for Case 1

Method	Class	Total OD Flow		Ratio
		Segment 1	Segment 2	Seg. 1/Seg. 2
VISUM	SOV	101.21	228.38	0.443
	HOV	8.67	18.25	0.475
TAPAS	SOV	104.67	236.01	0.443
	HOV	8.36	18.86	0.443

For Case 1, VISUM and TAPAS assign total flows to each segment in the same proportions for SOV and HOV flows, as shown in Table 4.4.

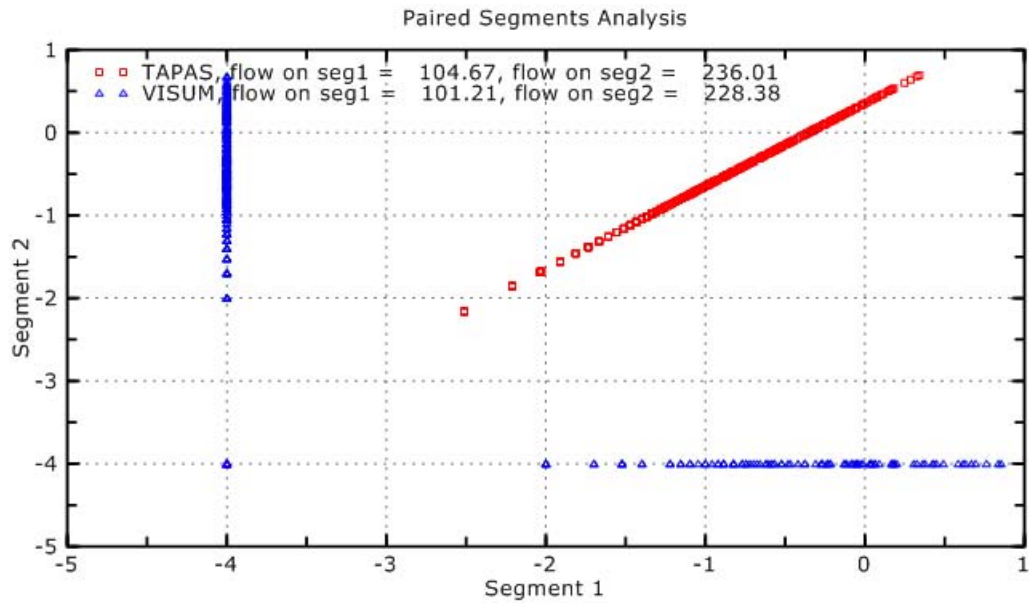


Figure 4.19a SOV OD flows on segments 1 and 2 by TAPAS and VISUM – log scale

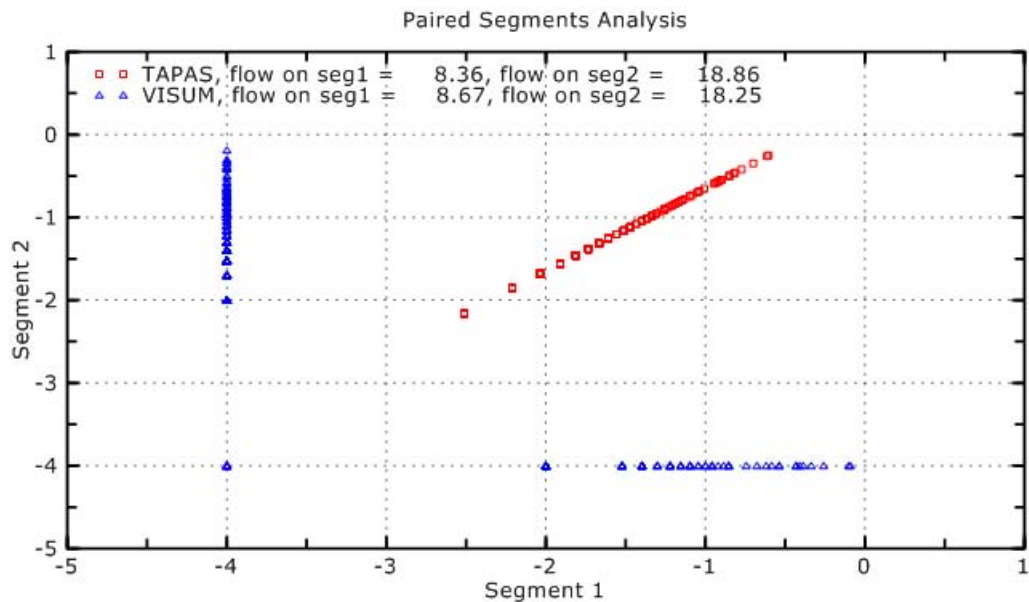


Figure 4.19b HOV OD flows on segments 1 and 2 for TAPAS and VISUM – log scale

For SOV flows, TAPAS found 404 OD pairs with flows distributed proportionately to each segment; VISUM finds 394 OD pairs with each flow distributed to only one segment. For TAPAS, the magnitude of the flows ranged from 0.003 to 4.95; for VISUM, they ranged from 0.01 to 7.14. For HOV flows, TAPAS found 323 OD pairs with flows distributed proportionately to each segment; VISUM found 315 OD pairs with each flow distributed to only one segment. For TAPAS, the magnitude of the flows ranged from 0.003 to 0.56; for VISUM, they ranged from 0.01 to 0.81. The VISUM flows shown above at (-4, -4) correspond to zero flows on the alternative segment to the segment with flow.

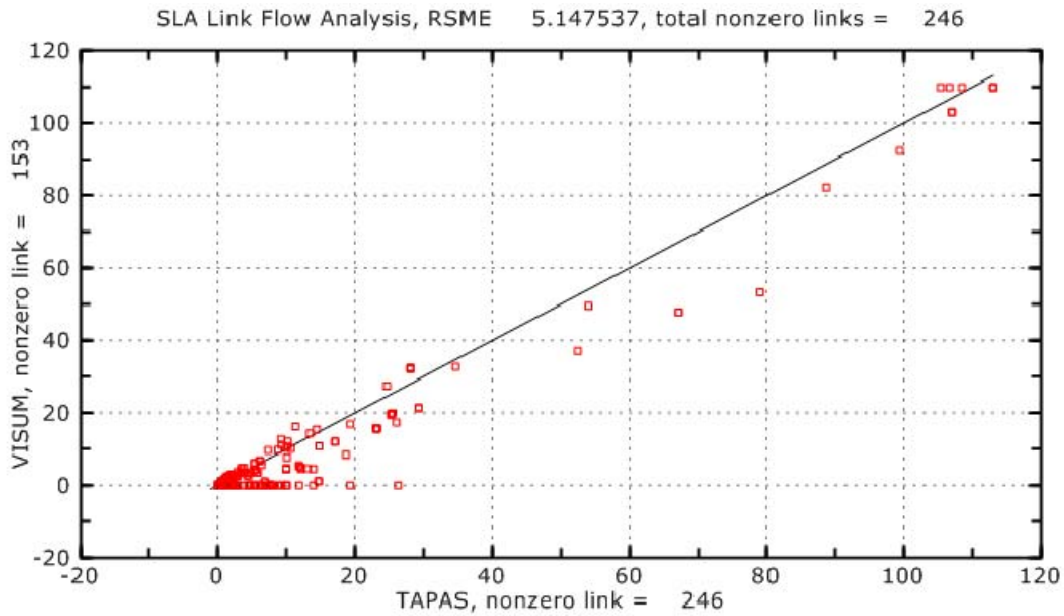


Figure 4.20a Total link flows on links for route flows using segment 1 – linear scale

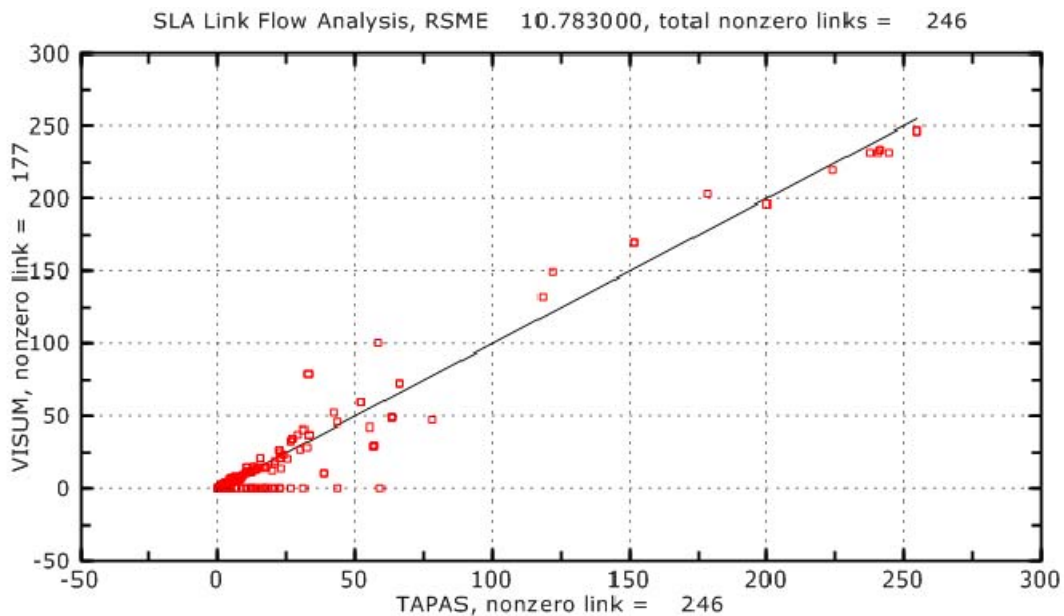


Figure 4.20b Total link flows on links for route flows using segment 2 – linear scale

The coordinates of each point show the total vehicle flow per period (SOV and HOV) on all routes that traverse both that link and the segment, as found by VISUM 10.0 and TAPAS.

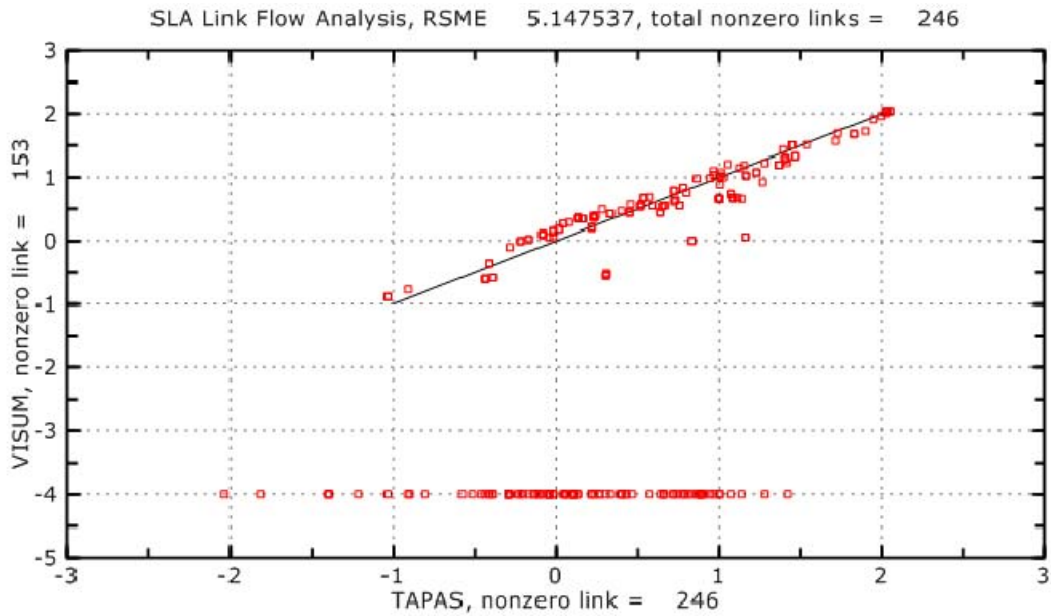


Figure 4.21a Total link flows on links for route flows using segment 1 – log scale

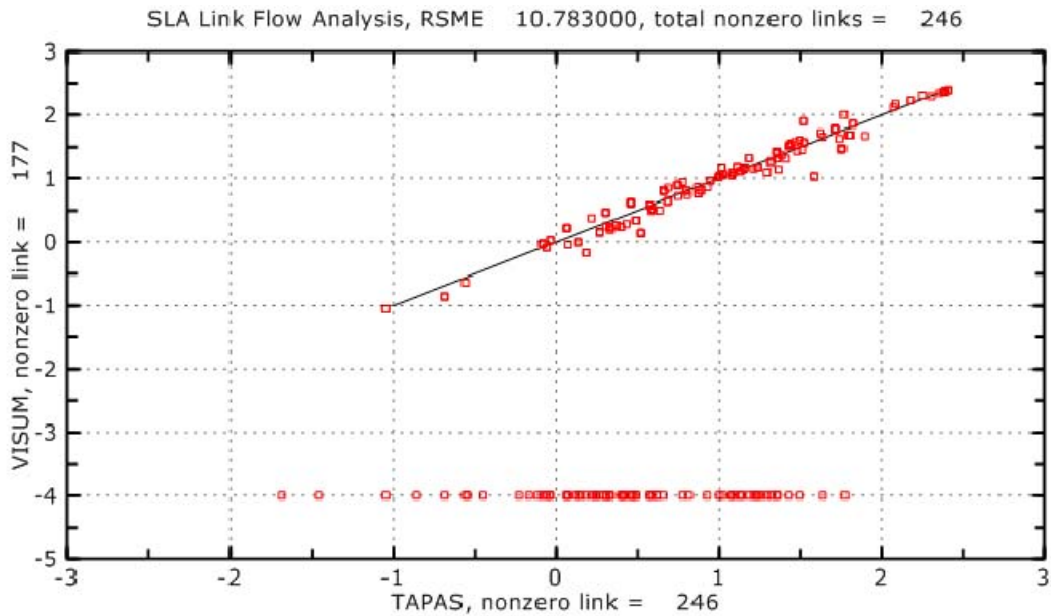


Figure 4.21b Total link flows on links for route flows using segment 2 – log scale

The log scale plots show numerous links with significant flows not found by the VISUM assignment; see the horizontal line of points opposite (-4) on the y-axis.

Case 2. The pair of alternative segments is shown in Figure 4.22.

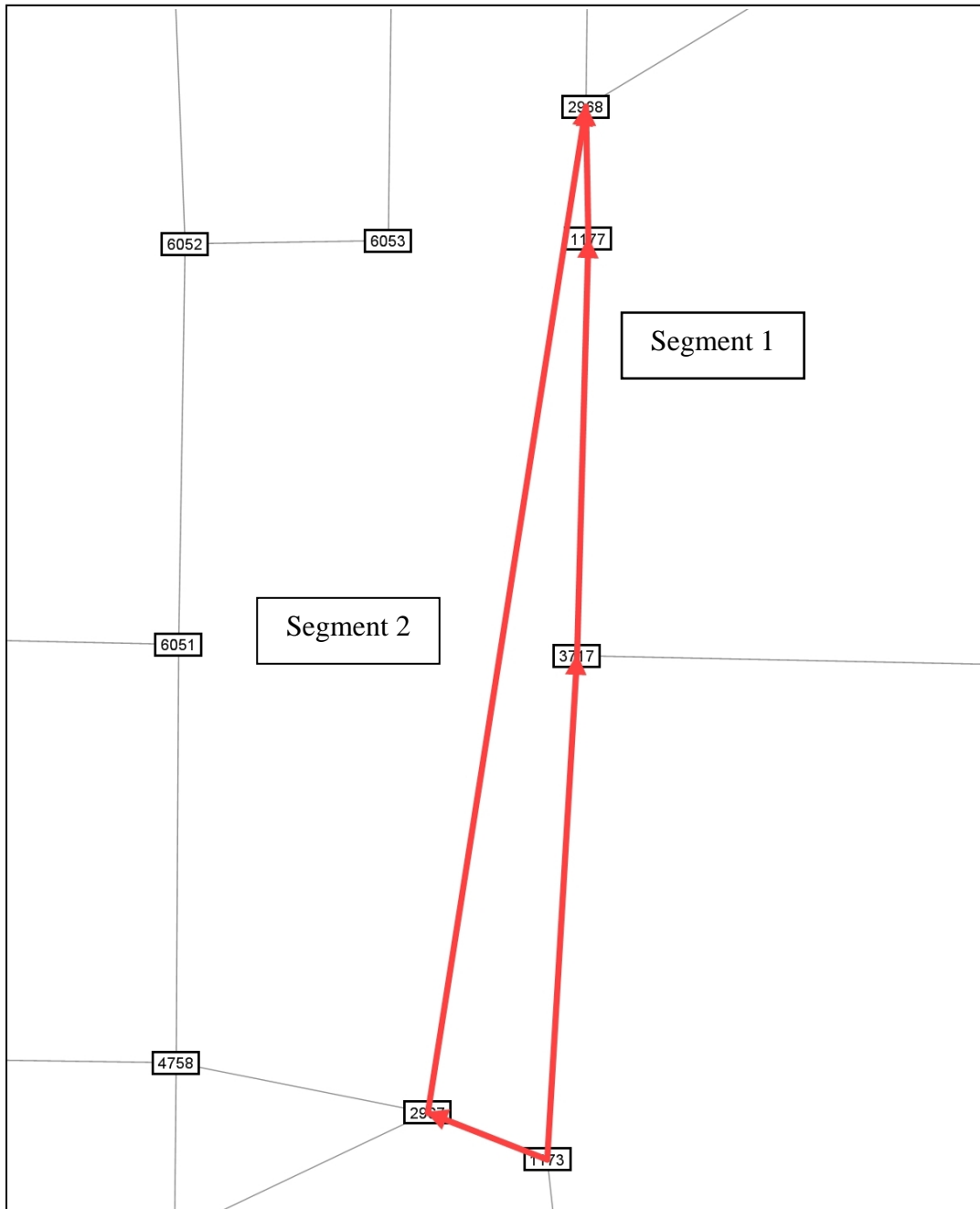


Figure 4.22 Segments 1 and 2 linking node 1173 to node 2968

Four sets of figures show the SOV and HOV route flows for this case:

1. SOV and HOV OD flows for segments 1 and 2 at linear and log scales (Figures 4.23 and 4.24);
2. total link flows for segments 1 and 2 at linear and log scales (Figures 4.25 and 4.26).

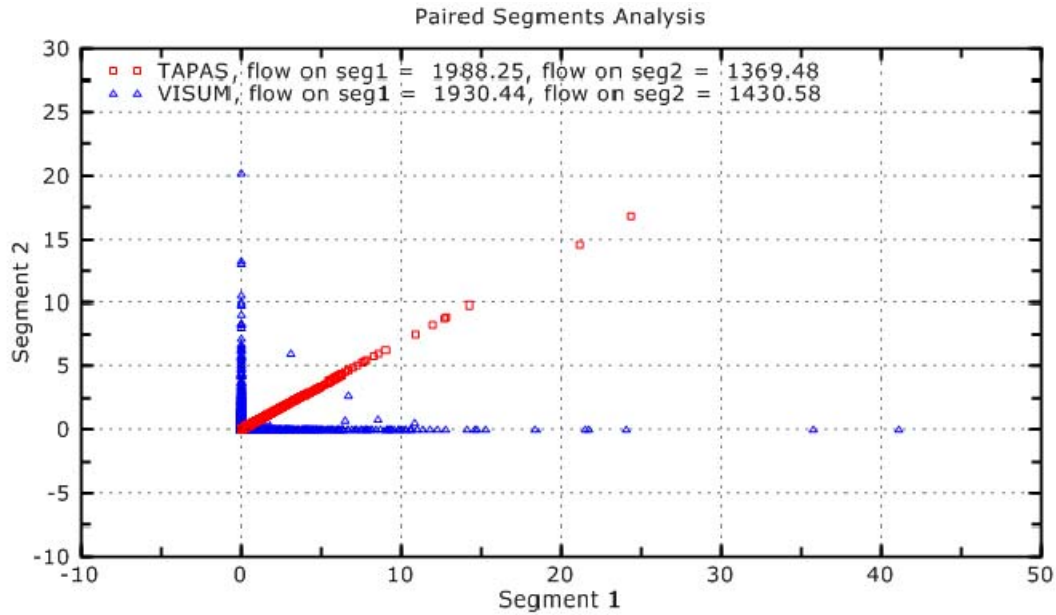


Figure 4.23a SOV OD flows on segments 1 and 2 by TAPAS and VISUM – linear scale

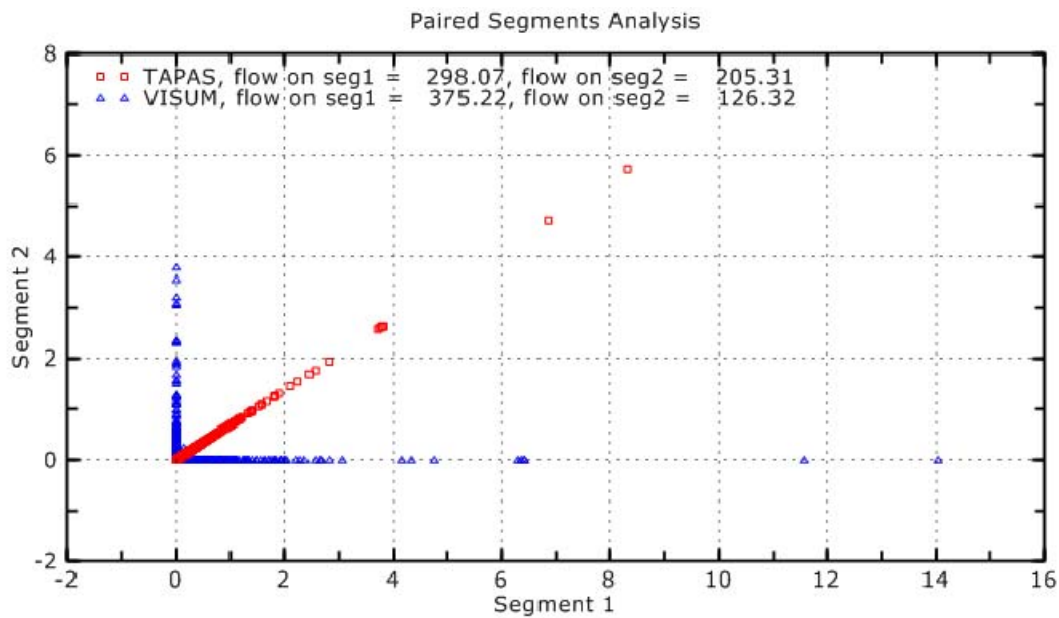


Figure 4.23b HOV OD flows on segments 1 and 2 for TAPAS and VISUM – linear scale

Table 4.5 Ratio of total segment flows for VISUM and TAPAS for Case 2

Method	Class	Total OD Flow		Ratio
		Segment 1	Segment 2	Seg. 1/Seg. 2
VISUM	SOV	1930.44	1430.58	1.3494
	HOV	375.22	126.32	2.9704
TAPAS	SOV	1988.25	1369.48	1.4518
	HOV	298.07	205.31	1.4518

As shown in Table 4.5, VISUM favors segment 1 for HOV flows, even though the costs are equal for both classes.

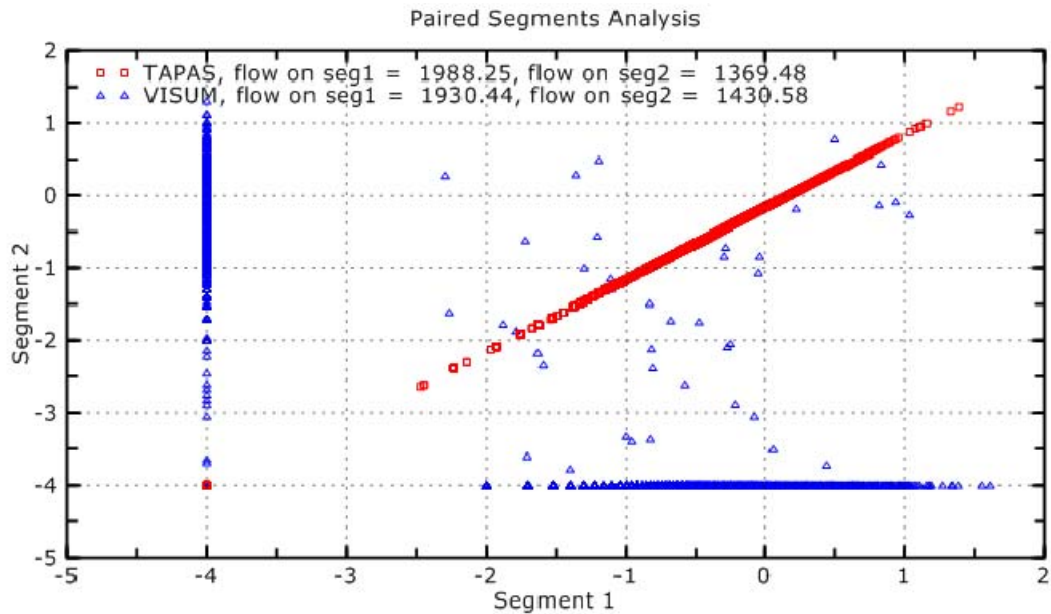


Figure 4.24a SOV OD flows on segments 1 and 2 by TAPAS and VISUM – log scale

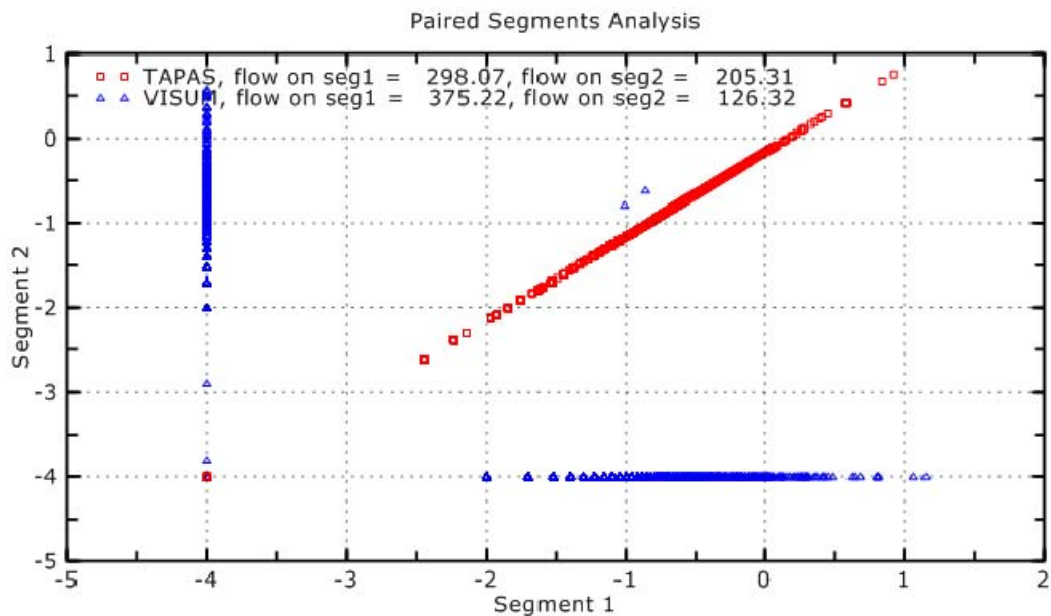


Figure 4.24b HOV OD flows on segments 1 and 2 by TAPAS and VISUM – log scale

For SOV flows, TAPAS found 5,446 OD pairs with flows distributed proportionately to each segment; VISUM found 5,301 OD pairs with each flow distributed to only one segment, and 43 OD pairs with flows distributed to both segments. For TAPAS, the magnitude of the flows ranged from 0.0023 to 24.35; for VISUM, they ranged from 1E-6 to 41.12. For HOV flows, TAPAS found 3,237 OD pairs with flows distributed proportionately to each segment; VISUM found 3,106 OD pairs with each flow distributed to only one segment, and 2 OD pairs with flows

distributed to both segments. For TAPAS, the magnitude of the flows ranged from 0.0025 to 8.31; for VISUM, they ranged from 0.0002 to 14.03. The TAPAS and VISUM flows shown above at (-4, -4) correspond to flows less than 1E-4.

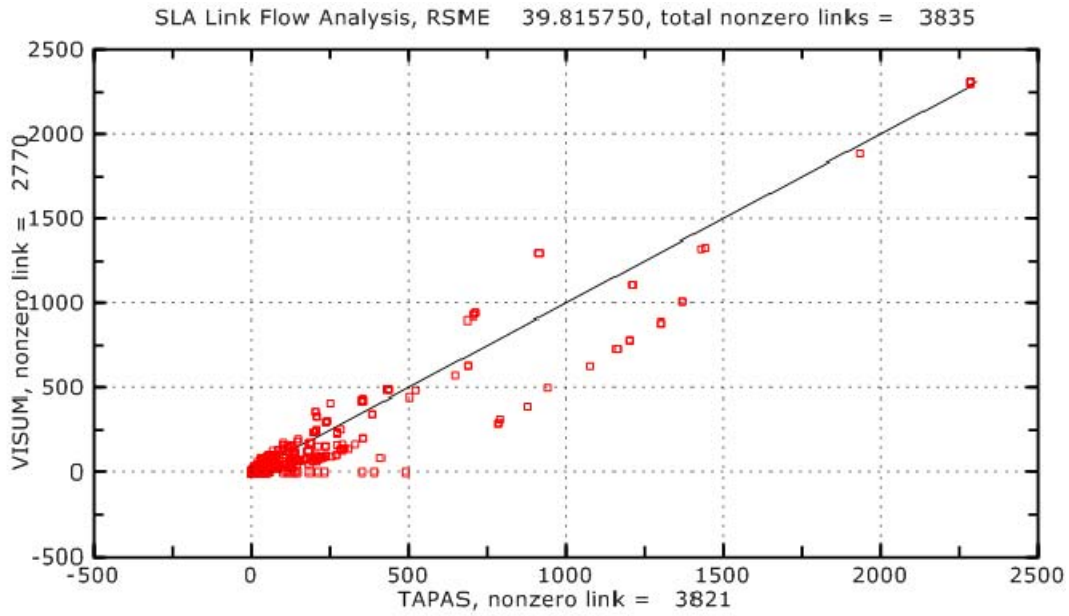


Figure 4.25a Total link flows on links for route flows using segment 1 – linear scale

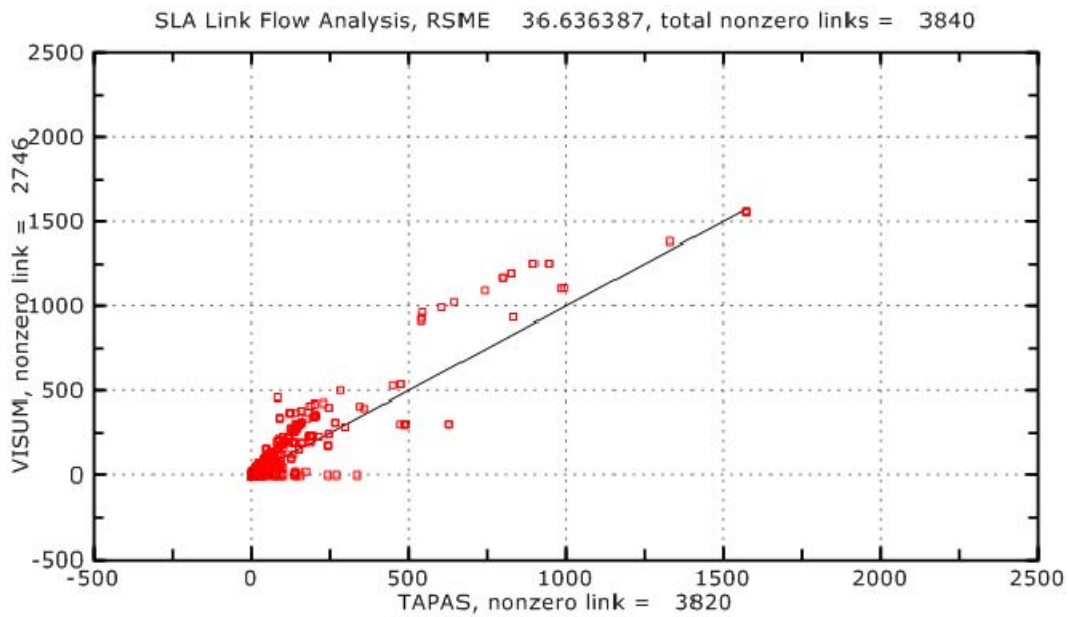


Figure 4.25b Total link flows on links for route flows using segment 2 – linear scale

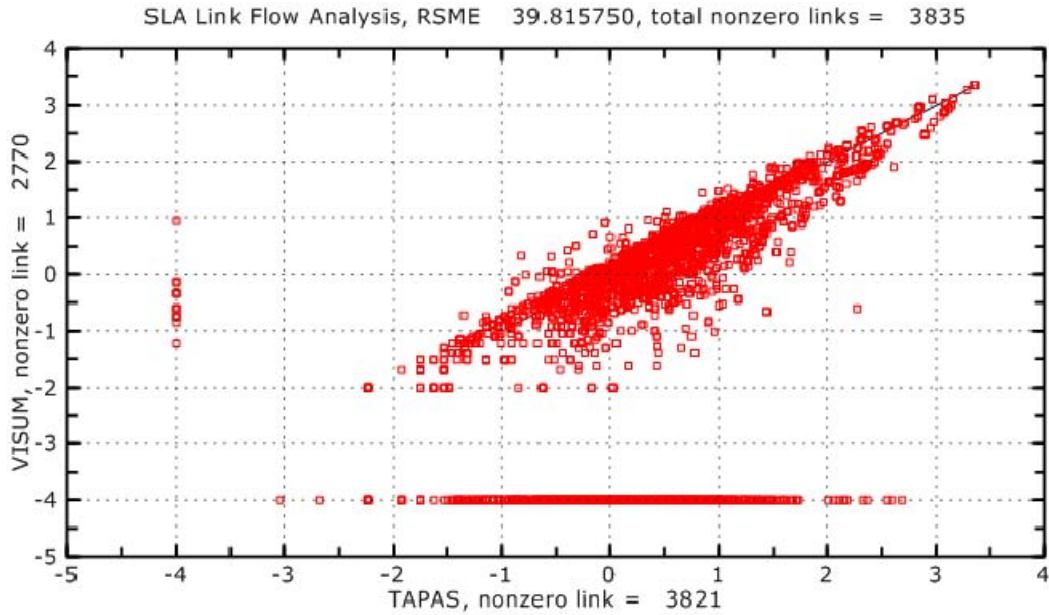


Figure 4.26a Total link flows on links for route flows using segment 1 – log scale

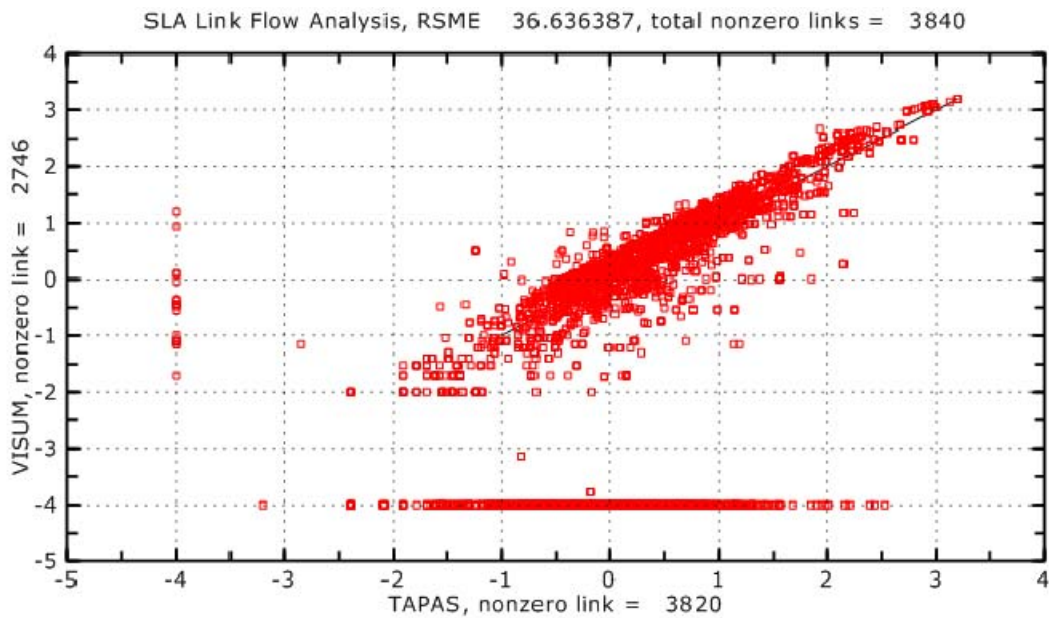


Figure 4.26b Total link flows on links for route flows using segment 2 – log scale

In the log plots, one may see that VISUM did not find link flows on over 1,000 links used by route flows passing over the PAS. Many of these links flows exceed 10 vehicles per period (1 on the log scale). On the linear plots, one may see that link flows over 500 vehicles per period are distributed quite differently on segments 1 and 2.

4.5.3 Conclusions of the collaborator

PTV is aware of the issues related to sparse route sets in its route-based assignment algorithm. These issues are being addressed in the LUCE assignment method, which is currently offered as a prototype in VISUM. Post processors for improved route proportionality are currently under development. In general, practitioners should keep this issue in mind when using matrix estimation methods.

VISUM also offers the Lohse assignment method, which is similar to the Frank-Wolfe method. The Lohse method may be used as an alternative to route-based assignment for matrix estimation.

4.6 Puget Sound Regional Council – RST International case study

4.6.1 Background, collaborator and study objective

Puget Sound Regional Council (PSRC) is the metropolitan planning organization for the Seattle-Tacoma metropolitan area in the State of Washington. Robert Tung, president of RST International, Inc., and a consultant to transportation planning agencies and engineering consultants in the Seattle area, has substantial experience with the PSRC travel forecasting models and the application of EMME for forecasting travel. He agreed to collaborate with the project team, and contribute his valuable experience.

Version 9.5 of the EMME software system, released in September 2003 and licensed to the Washington State Department of Transportation, was used for this collaboration. The test network and trip matrices were converted from the 2006 PSRC PM peak period model. Because the research code version of TAPAS can assign only two classes, the 11 classes of travelers in the PSRC model were collapsed to two, single occupancy vehicles (SOV) and high occupancy vehicles (HOV). Trip matrices pertaining to the three-hour peak period were converted to one-hour matrices by applying the following peak-period factors: 0.33 for SOV, and 0.40 for HOV. The total SOV flow is 457,504.45 vehicles per hour; the total HOV flow is 177,402.77 vehicles per hour. Hence the total flow is 634,907.22.

For the network used in this analysis with TAPAS, the number of zones is 1,077; the network has 6,081 nodes and 18,738 links. The 660 HOV links in the TAPAS network were coded with travel times of $1E+10$ to prevent their use for SOV travel. The multi-modal network coding in EMME was converted to cost based coding, and a large hypothetical cost was imposed on HOV only links to prevent SOV trips from using them. In the analyses presented in this case study, only links with both SOV and HOV flows are considered.

The linear approximation (LA) option in EMME has been generally used for assignments of PSRC trip matrices. In this study, the two-class option of EMME was used. The Relative Gap for the EMME solutions was set to $1E-4$. A smaller RG would have been prohibitively time consuming and its benefits only marginal in terms of additional refinement of the solution. Moreover, a Relative Gap of $1E-4$ is much smaller than the precision of $1E-2$ used in typical applications of planning agencies in the Puget Sound region. The Relative Gap of the TAPAS solution is $2.9E-10$.

The objective of this case study is to explore the differences between a typical practitioner application, with a practical level of convergence as found by a widely applied software system, and an ultra high precision solution from research software. A secondary consideration concerns the uniqueness of the route flow solution.

Presented below are plots comparing the TAPAS and EMME solutions for three links for the two user classes for OD flows and for total link flows.

4.6.2 Description of case study findings

From ten select link analyses prepared by Robert Tung, three were selected for presentation in this report. These links form the I-5 Ship Canal Bridge, and are shown in Figure 4.27.

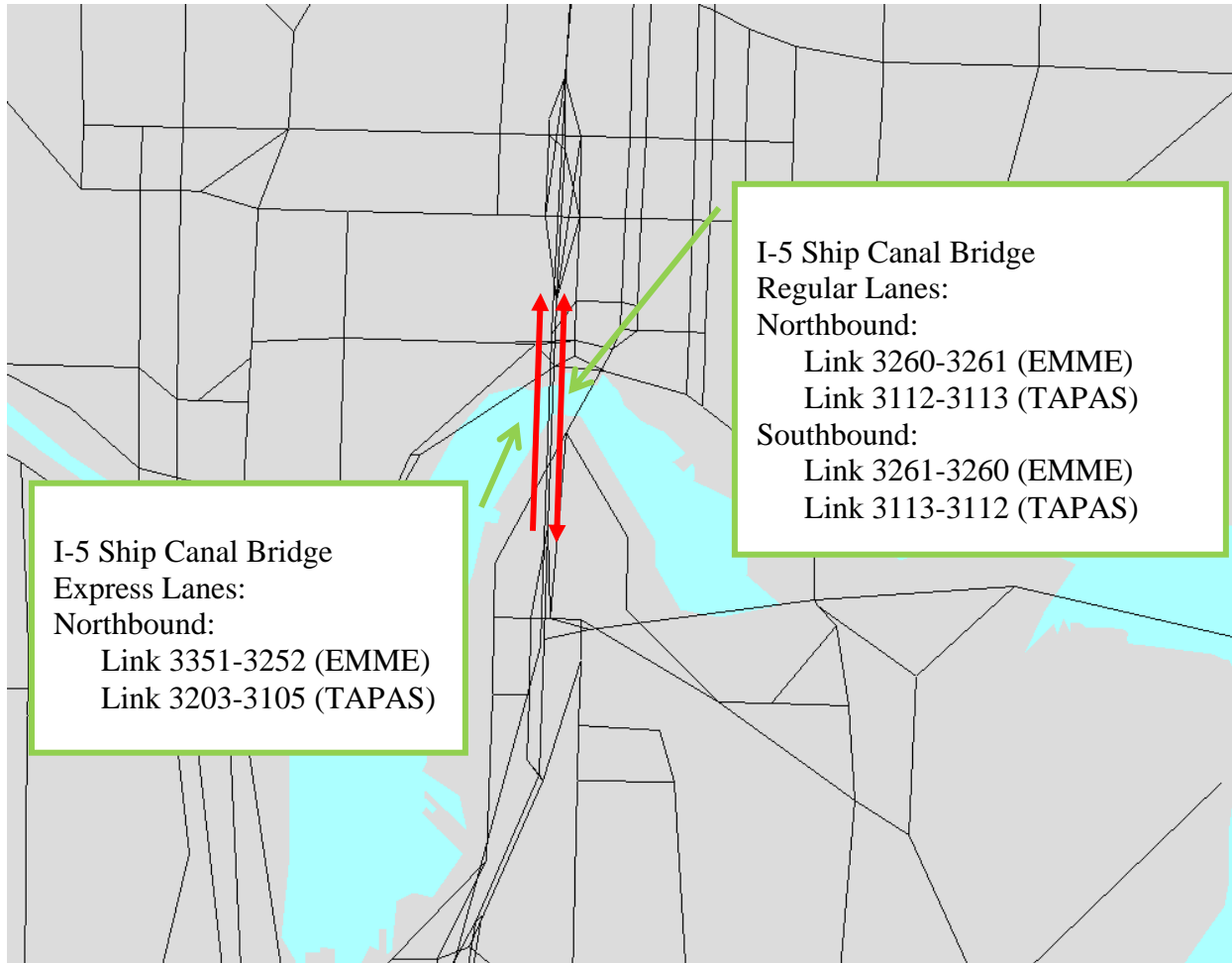


Figure 4.27 Location of the three selected links on the I-5 Ship Canal Bridge

These links were chosen for presentation to illustrate examples for a major north-south freeway through the Seattle-Tacoma region. A major alternative or bypass route, I-405, also serves long-distance traffic, suggesting that choices are available to drivers during the highly congested afternoon peak period.

Results for three links are presented: NB and SB regular lanes and NB express lanes operating in the afternoon peak period. For each link, the following plots are shown: SOV linear and log OD flows; HOV linear and log OD flows; linear and log total link flows. Plots are sized to be easily compared whenever possible.

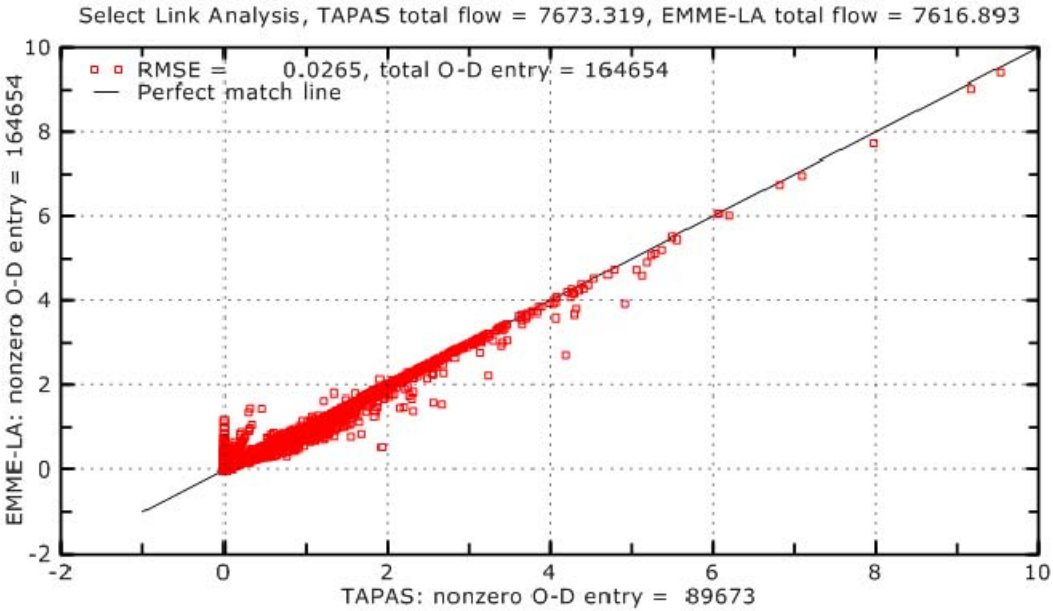


Figure 4.28 Northbound SOV OD flow, I-5 Ship Canal Bridge, regular lanes – linear scale (One high volume OD pair (939, 950) with 63.722 vehicles per hour is omitted from the plot.)

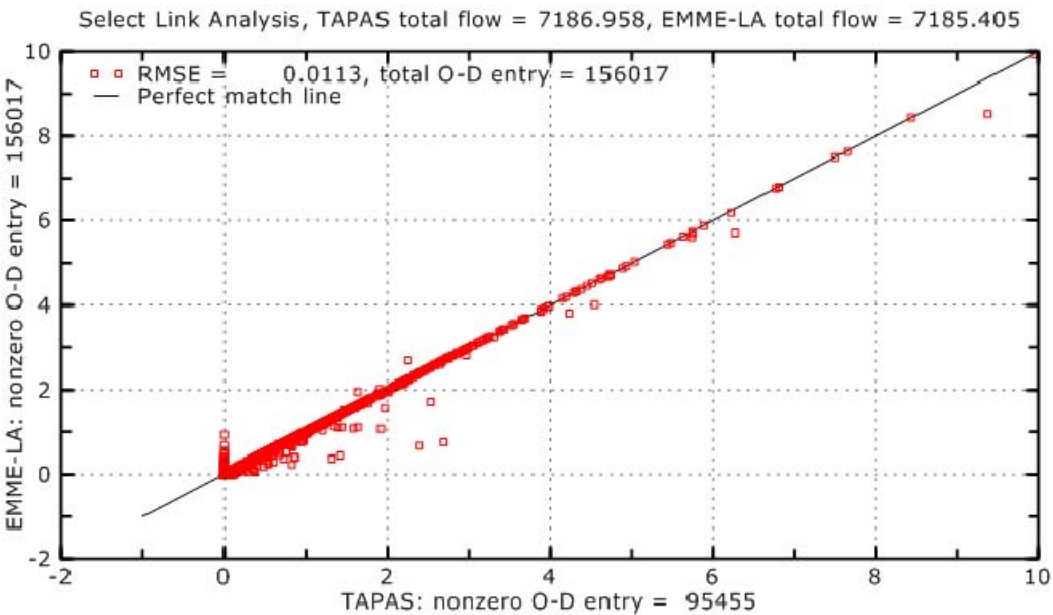


Figure 4.29 Southbound SOV OD flow, I-5 Ship Canal Bridge, regular lanes – linear scale (One high volume OD pair (950, 939) with 136.689 vehicles per hour is omitted from the plot.)

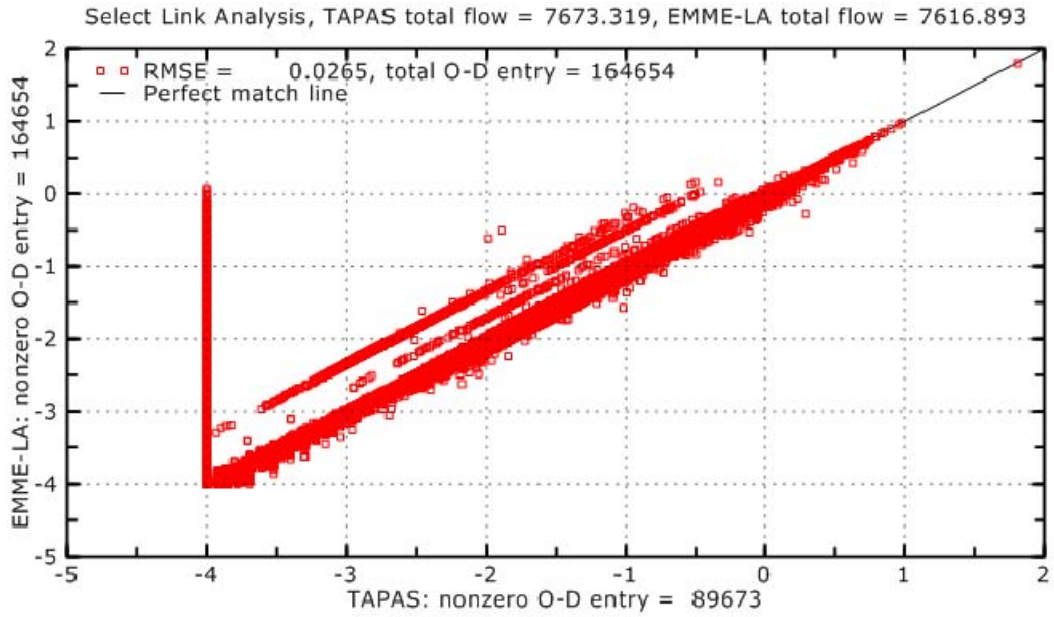


Figure 4.30 Northbound SOV OD flow, I-5 Ship Canal Bridge, regular lanes – log scale

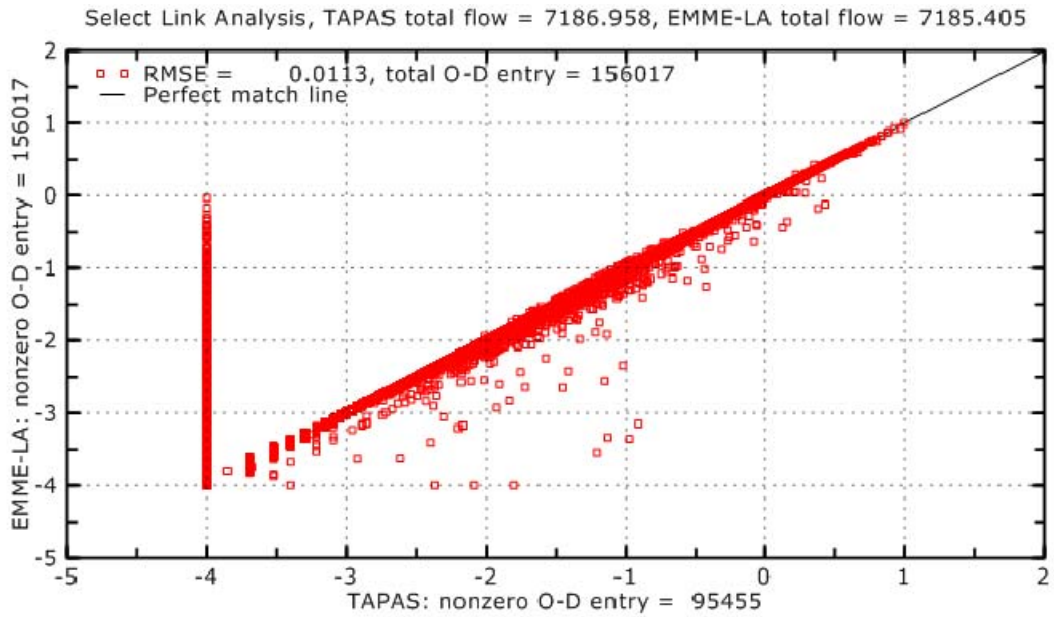


Figure 4.31 Southbound SOV OD flow, I-5 Ship Canal Bridge, regular lanes – log scale (One high volume OD pair (950, 939) with a log value of 2.14 is omitted from the plot.)

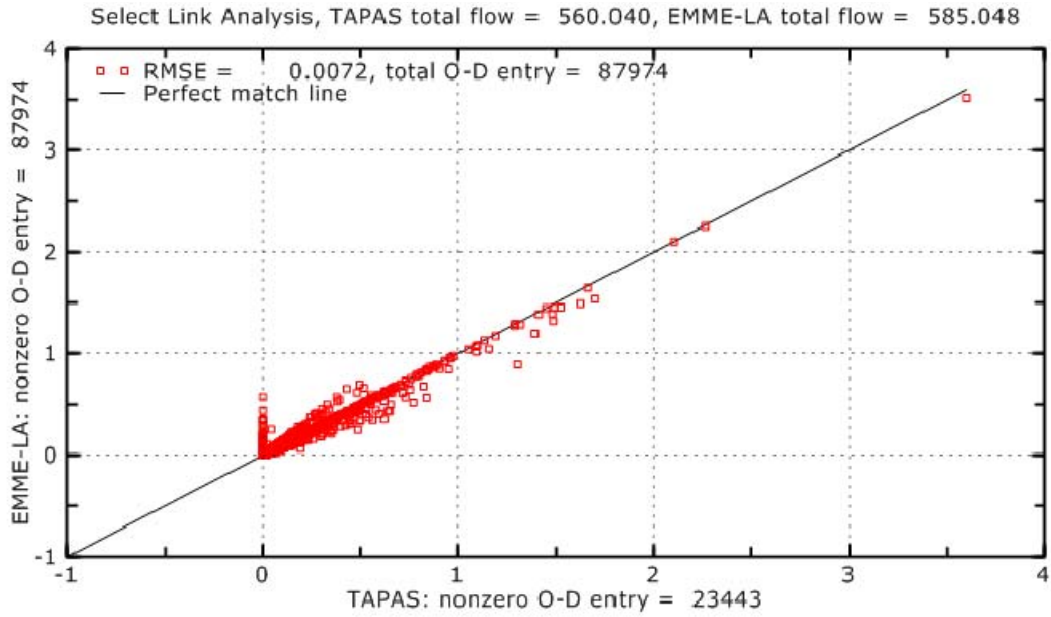


Figure 4.32 Northbound HOV OD flow, I-5 Ship Canal Bridge, regular lanes – linear scale

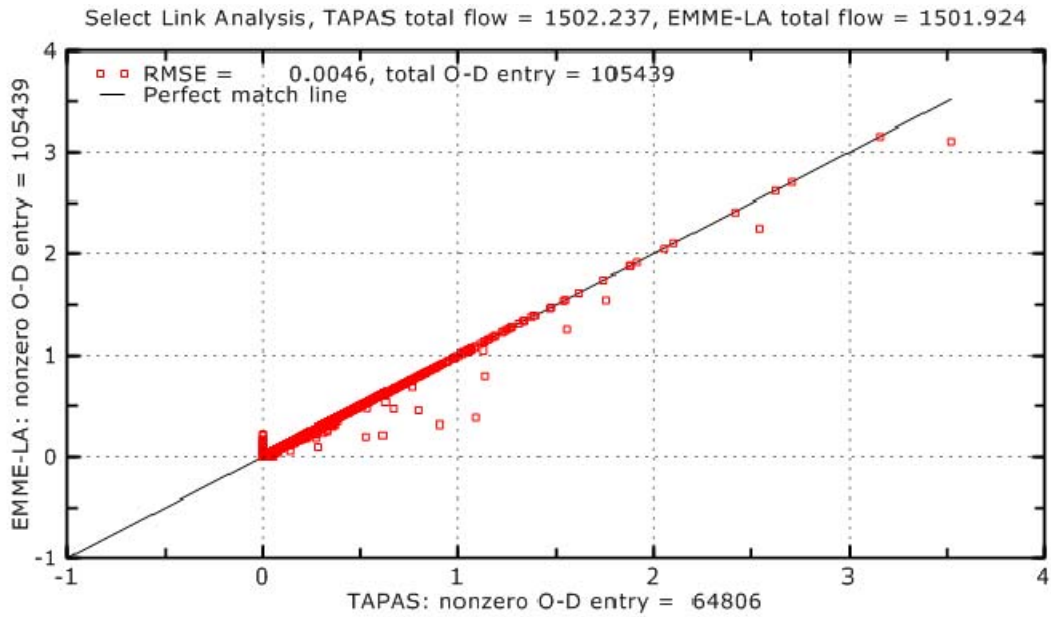


Figure 4.33 Southbound HOV OD flow, I-5 Ship Canal Bridge, regular lanes – linear scale

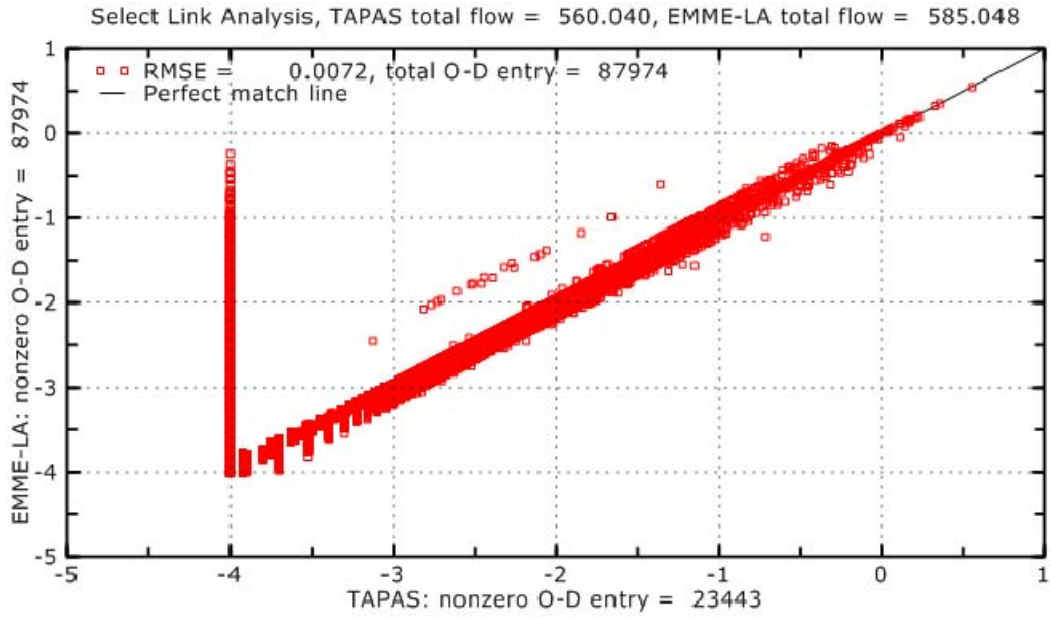


Figure 4.34 Northbound HOV OD flow, I-5 Ship Canal Bridge, regular lanes – log scale

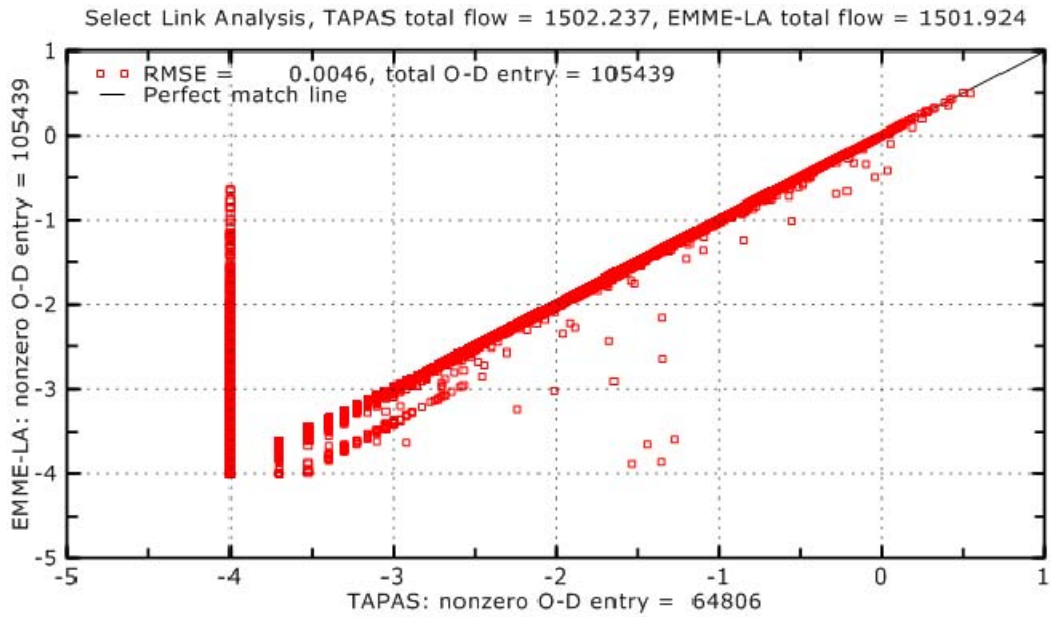


Figure 4.35 Southbound HOV OD flow, I-5 Ship Canal Bridge, regular lanes – log scale

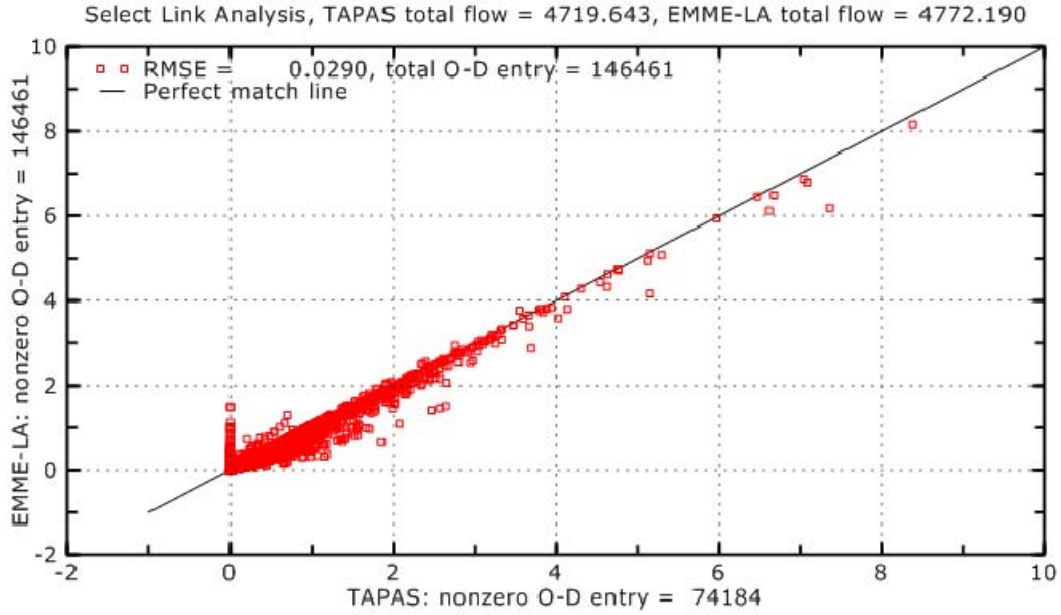


Figure 4.36 Northbound SOV OD flow, I-5 Ship Canal Bridge, express lanes – linear scale (One high volume OD pair (939, 950) with 28.42 vehicles per hour is omitted from the plot.)

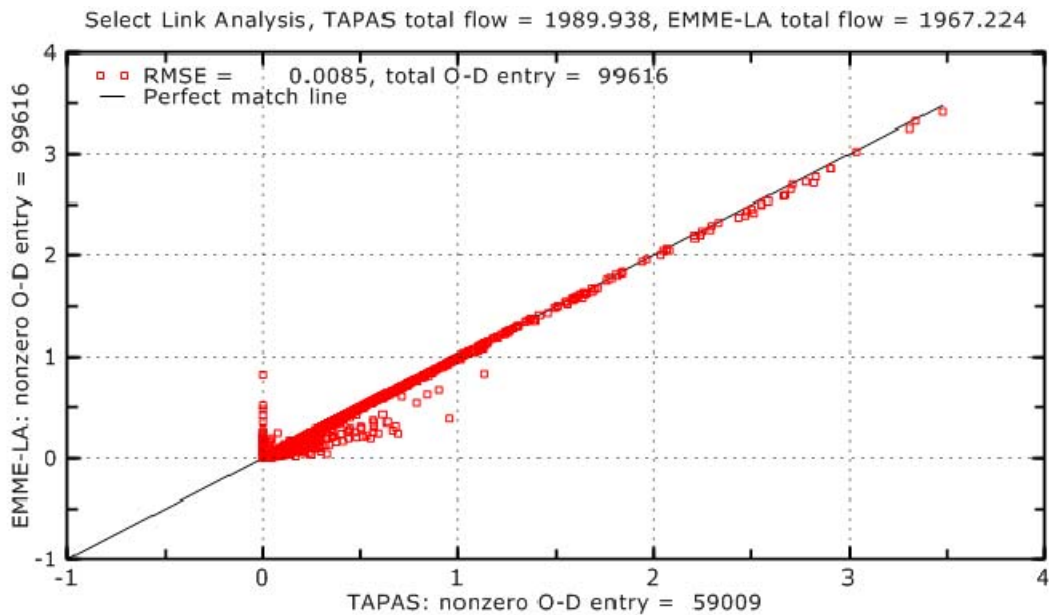


Figure 4.37 Northbound HOV OD flow, I-5 Ship Canal Bridge, express lanes – linear scale

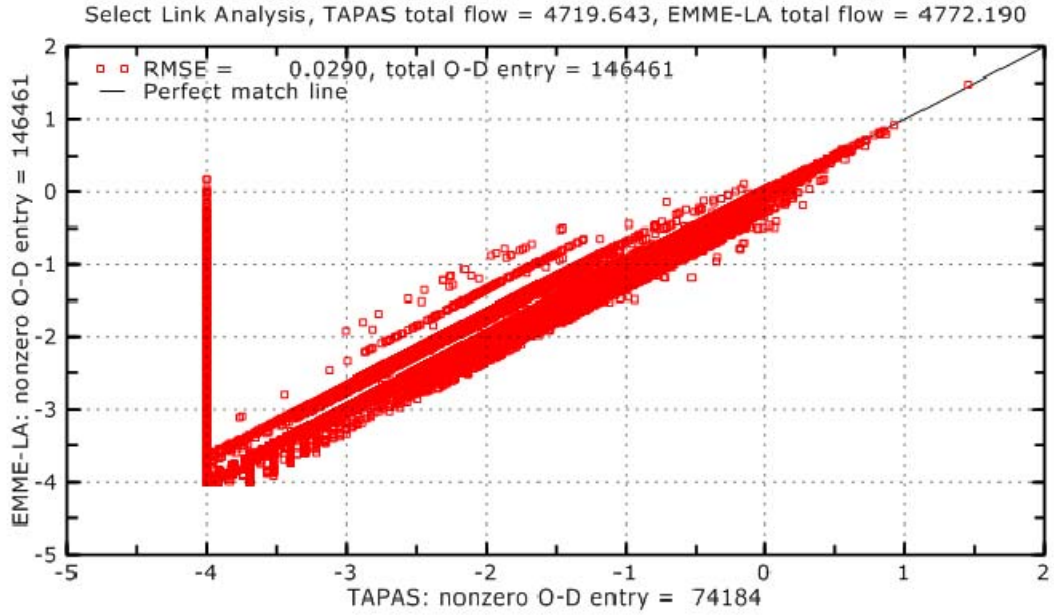


Figure 4.38 Northbound SOV OD flow, I-5 Ship Canal Bridge, express lanes – log scale

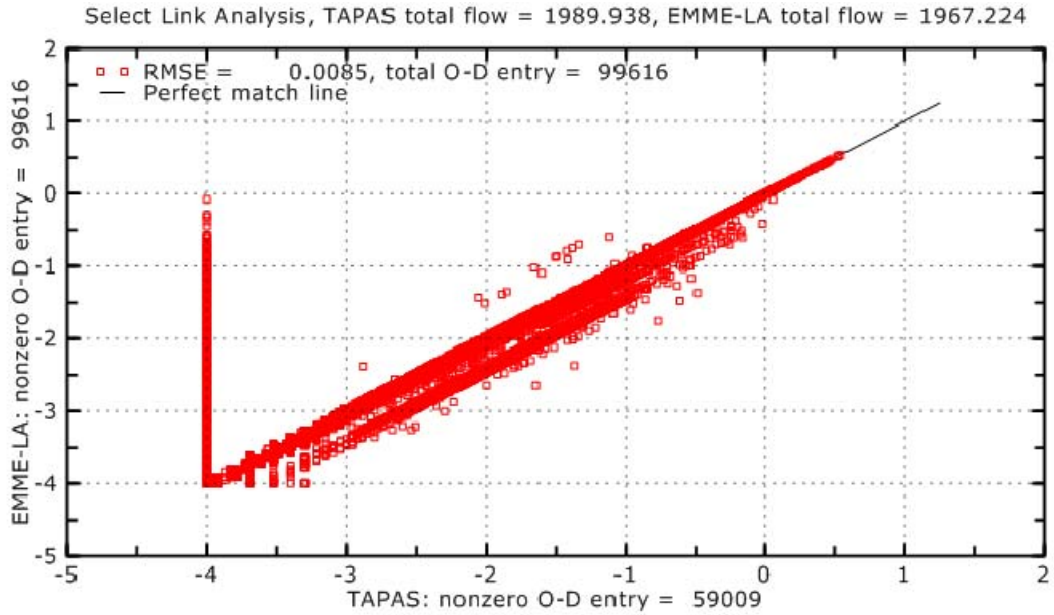


Figure 4.39 Northbound HOV OD flow, I-5 Ship Canal Bridge, express lanes – log scale

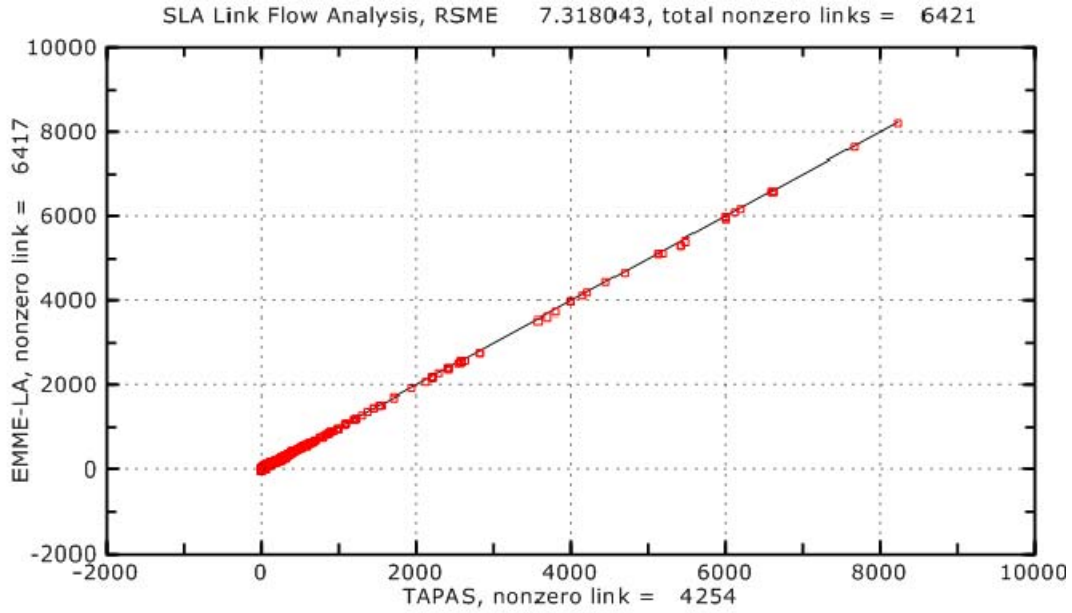


Figure 4.40 Northbound total link flow, I-5 Ship Canal Bridge, regular lanes – linear scale

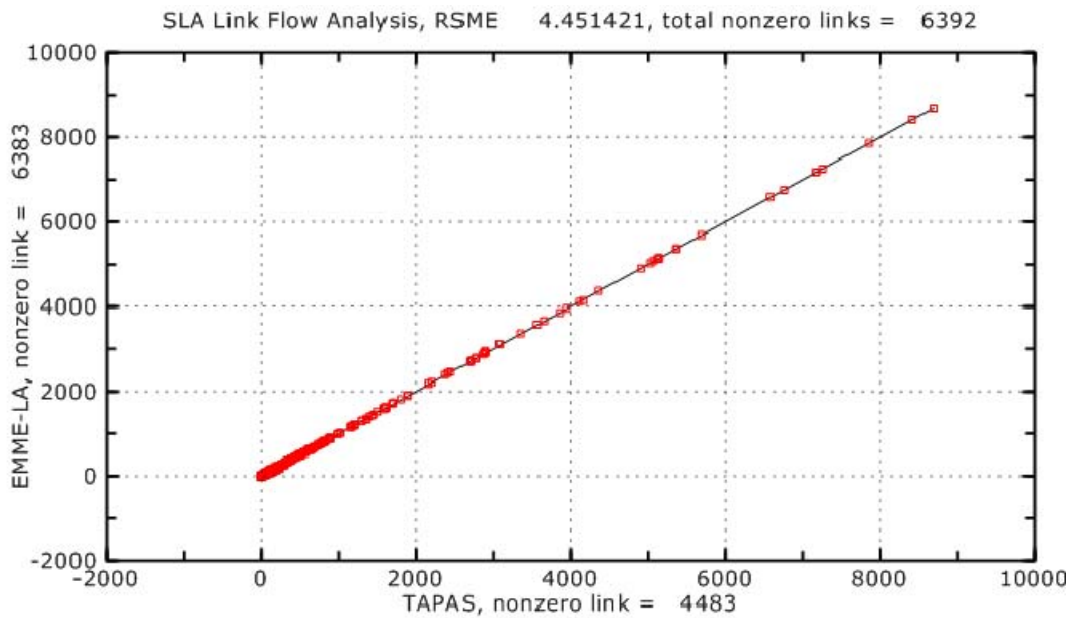


Figure 4.41 Southbound total link flow, I-5 Ship Canal Bridge, regular lanes – linear scale

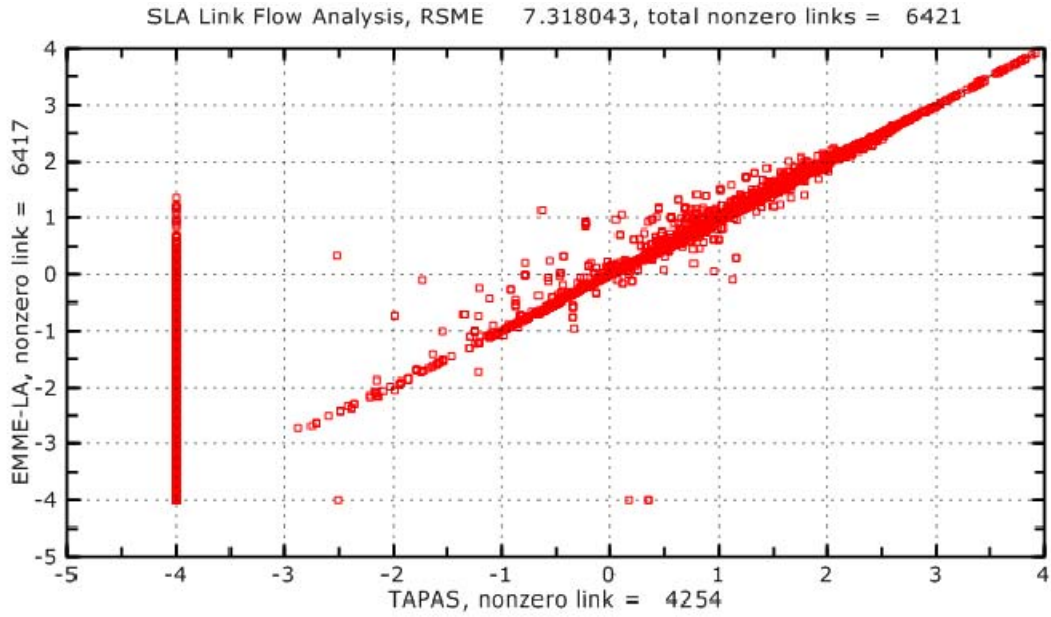


Figure 4.42 Northbound total link flow, I-5 Ship Canal Bridge, regular lanes – log scale

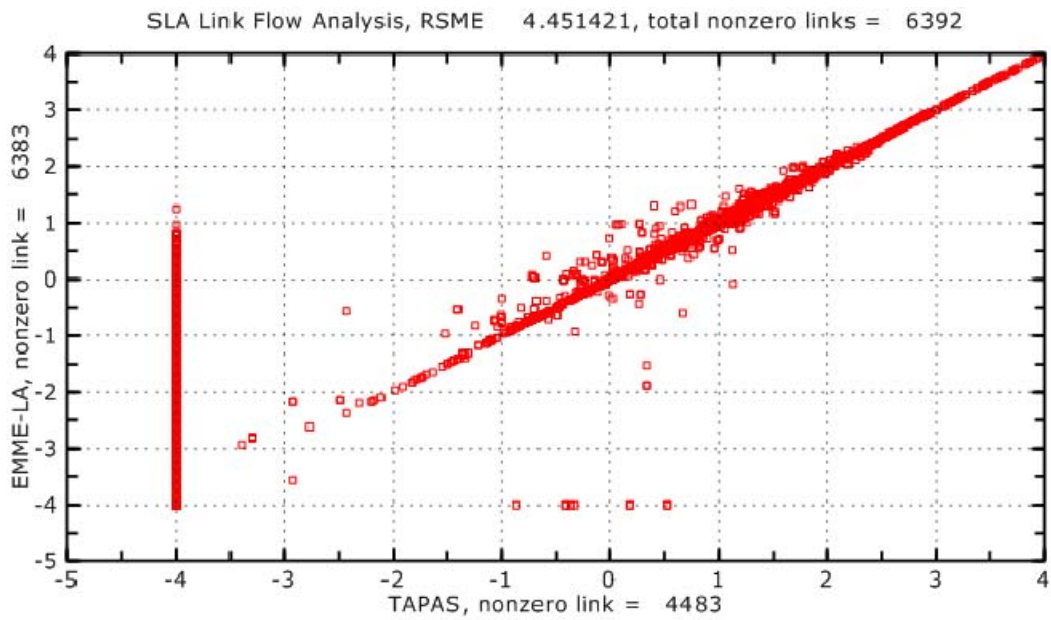


Figure 4.43 Southbound total link flow, I-5 Ship Canal Bridge, regular lanes – log scale

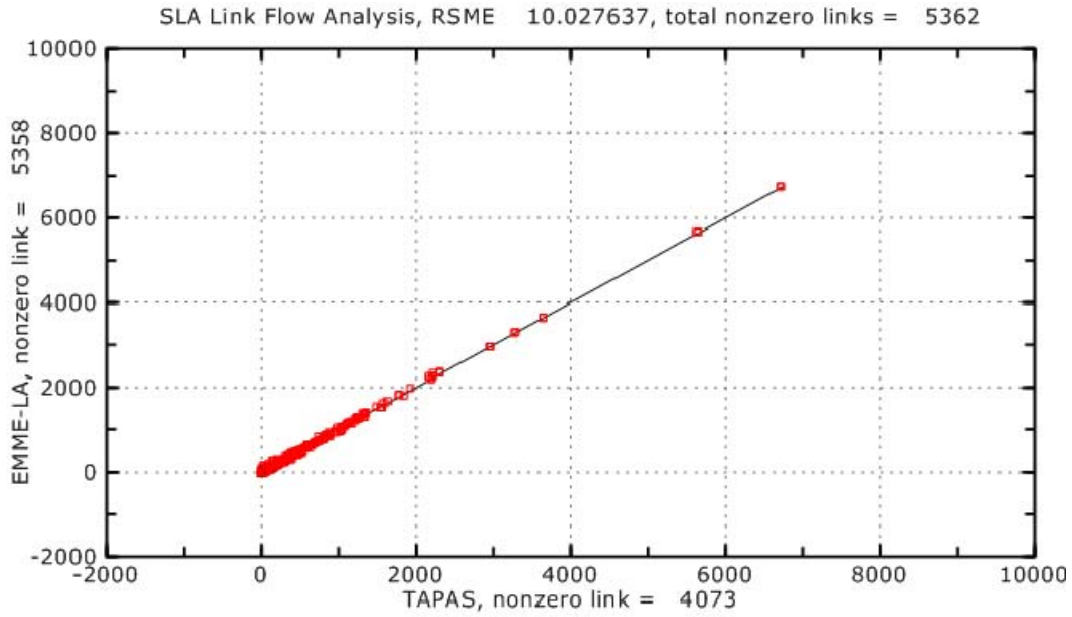


Figure 4.44 Northbound total link flow, I-5 Ship Canal Bridge, express lanes – linear scale

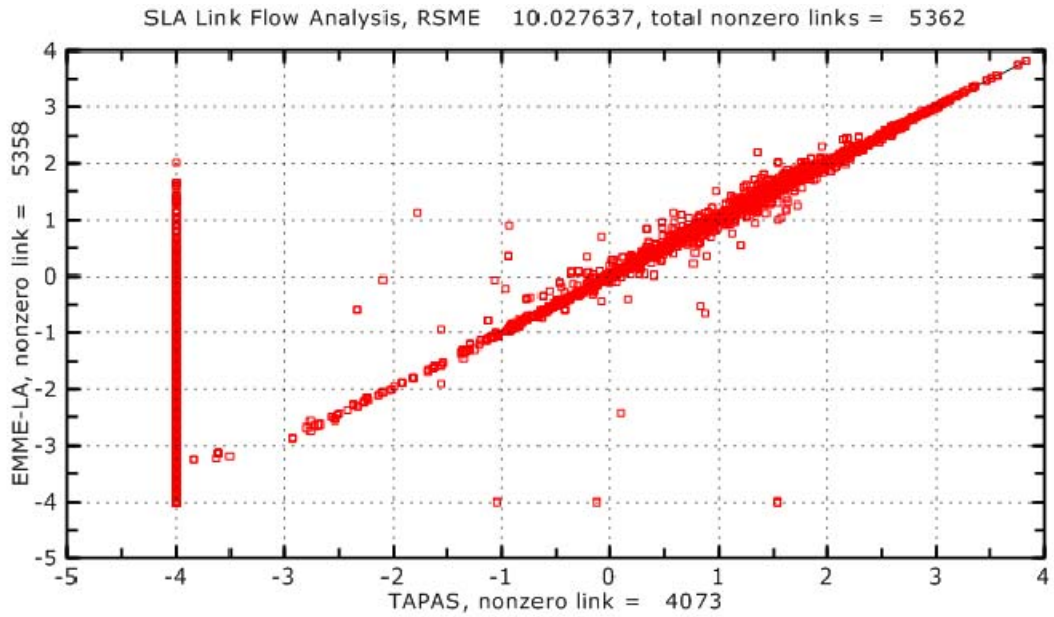


Figure 4.45 Northbound total link flow, I-5 Ship Canal Bridge, express lanes – log scale

4.6.3 Conclusions of the collaborator

The plots shown above are the basic building blocks for comparing both OD flows by class and total link flows for each selected link. Although the purpose of the scatter plots is to show the general relation between flows identified by TAPAS and EMME, they are also useful for detecting the existence of any large outliers. For this study generally, no such outliers are found. All data points generally fall on the perfect match line or very close to it. This high correlation is also indicated by the low RMSE values.

However, there are two notable differences in the results:

1. EMME generates more positive route and link flows than TAPAS. The number of positive OD flows found by EMME can be as high as twice the number found in the more highly converged TAPAS solution, as shown on the y-axis of the plots.
2. Although the scatter plots do not show significant outliers, notable differences in the EMME and TAPAS flows may be observed for very small flows. These small differences can be closely examined in the log plots, where EMME shows positive flows on the log scale ranging from -4 up to +1, corresponding to flows of 0.0001 to 10.0. Note that flows less than 0.0001 are plotted at the 0.0001 point. As the corresponding flows in the TAPAS solution are zero, these OD pairs form a vertical line along the Y axis indicating EMME produces many more positive flows, even though they are very small. These flows are largely residual non-equilibrium flows generated by EMME's linear approximation (Frank-Wolfe) algorithm, which are not completely eliminated before the solution process is terminated.

In addition to the scatter plots, the flow differences may be also displayed in GIS-based maps using UfosNet; an example is shown below as Figure 4.46. In this map, the red lines show links where EMME assigned slightly more trips than TAPAS. Conversely, black lines show links where TAPAS assigned slightly more trips than EMME. The widths of those lines are proportional to the magnitude of the small differences in link flows found by the two algorithms. By reviewing these plots, one can see a pattern of differences along routes adjacent to the I-5 corridor, which is another illustration of how the two assignment methods differ in finding the user equilibrium solutions. Evidently, the differences are relatively small and highly localized. The differences observed here may not have a major influence in an alternatives analysis for regional planning purpose, and may be neglected in most cases. Of course, for other links or corridors, the differences could be larger.

In conclusion, to demonstrate and validate the capabilities of TAPAS, this study compared TAPAS to a Frank-Wolfe algorithm embedded in the popular modeling tool EMME, using the PSRC model as the test platform. As indicated above, the results from these two algorithms are very similar by both statistical measures and visual comparisons. Nevertheless, significant differences may be observed in the number of positive OD and link flows with small values. Without further tests, these differences can be largely attributed to the high precision of TAPAS in computing the user-equilibrium solution.

User-equilibrium (UE) assignment is a critical part of a travel forecasting procedure. Uniqueness of route flows and efficiency in solving UE problems have significant impacts on planning and operations analyses. It is well known that the traditional Frank-Wolfe algorithms are slow to converge; moreover, they may not produce route flows that are unique, and therefore are not comparable among scenarios. Transportation practitioners and researchers have a highly vested interest in finding more efficient algorithms and more precise solutions. A few new algorithms have emerged in recent years that have claimed faster convergence. However, until TAPAS none has tackled both efficiency and uniqueness at the same time. In this regard, TAPAS is perhaps one of the best examples of a next generation UE algorithm.

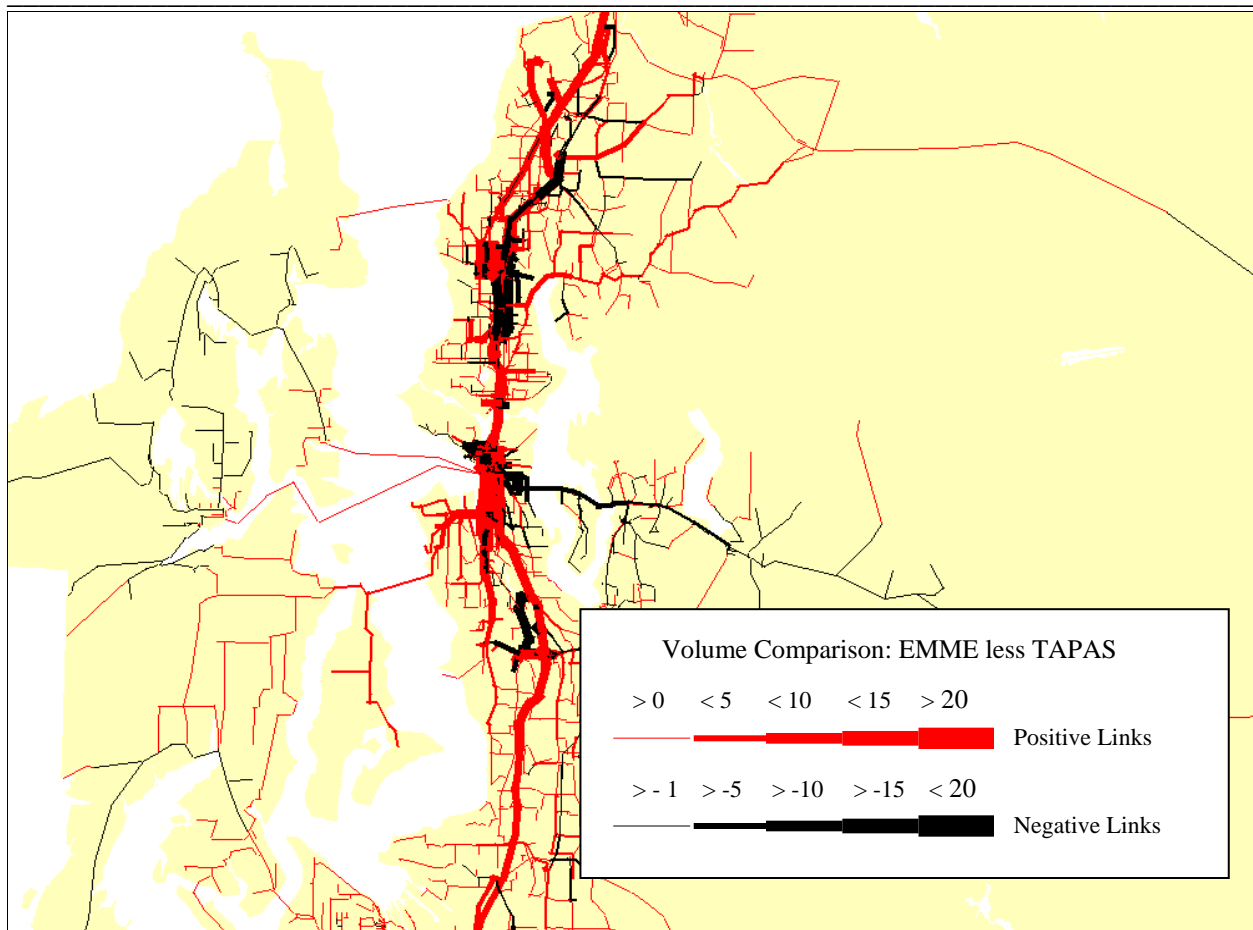


Figure 4.46 I-5 Ship Canal Bridge express lanes northbound: select link flow comparison

4.7 Ohio Department of Transportation case study

4.7.1 Background, collaborator and study objective

The Ohio Department of Transportation (ODOT) regularly conducts travel forecasting studies for the evaluation of proposed road improvements. The project considered in this case study is a freeway extension project in Northwest Ohio. This project would replace an existing two-lane roadway, US 24, between Toledo, OH and Ft. Wayne, IN, with a four-lane divided facility. The assignment algorithms being tested were the CUBE user-equilibrium (Frank-Wolfe) assignment algorithm and the TAPAS algorithm.

The Department used its standard User Benefits program to compute the Build – No Build benefits for the project for two classes of vehicles, cars and trucks. The User Benefits program aggregates benefits resulting from crashes, travel time, fuel, other operating benefits and tolls. In past studies, ODOT has found that a highly converged assignment is required to calculate the difference in User Benefits between two alternatives, such as Build and No Build. Several hundred iterations of the Frank-Wolfe algorithm implemented in CUBE are necessary to solve for a user-equilibrium assignment for the required level of convergence, which is a time consuming step when working on many projects. The Department typically uses 500 iterations in an effort to assure adequate convergence, with a target Relative Gap of $1E-4$. In this case study, the assignments terminated at Relative Gaps of $8E-5$ for the No Build case and $1E-6$ for the Build case. The assignment solution produced by TAPAS has a Relative Gap of $5.3E-17$ for the No Build 2030 case. The assigned networks were moderately congested.

The Northwest Ohio model utilized for this case study is a two-class assignment of cars and trucks to the network. The model implemented for 2010 has 395 zones, 9,289 nodes and 21,112 links. For 2010, the total number of vehicles in the car class is 2,443,532 vehicles per day; the number of vehicles in the truck class is 313,904 passenger car equivalents per day. The model implemented for 2030 has 396 zones, 9,312 nodes and 21,250 links. For 2030, the total number of vehicles in the car class is 2,692,750 vehicles per day; the number of vehicles in the truck class is 382,598 passenger car equivalents per day.

The case study was performed by Rebekah Anderson, Transportation Engineer.

The objective of this case study was to compare the output from ODOT's CUBE software system with the output from TAPAS for a two-class assignment. Unlike the other case studies that focused on select link and related analyses, this study was concerned with the effect of a more precisely converged assignment on the values of summary measures for use in scenario analyses.

In the usual procedure applied by ODOT, passenger car equivalent (PCE) values vary by link in an effort to capture the effect of geometric features on operating characteristics. In the manner applied by ODOT, the use of variable PCE values is not compatible with the integral in the objective function in the standard user-equilibrium formulation. For this reason, a single PCE value of 1.62 passenger cars per truck was used in converting truck flows to PCE flows.

4.7.2 Description of case study findings

Table 4.6 below shows the costs for each scenario based on the solutions produced by CUBE and TAPAS for the 2030 analysis year. In both cases, the TAPAS solution shows lower total costs, implying that it is more precisely converged. Table 4.7 shows the benefits of the project by the two solutions converted to present values. These results suggest that the benefits from this project may be somewhat overstated if the solutions are not adequately converged.

Table 4.6 Costs for 2030 by scenario and assignment algorithm

	No Build		Build	
	Cube	TAPAS	Cube	TAPAS
Crash	\$ 1,691,182,733	\$ 1,692,755,478	\$ 1,659,395,796	\$ 1,662,410,132
Time	\$ 4,754,061,410	\$ 4,744,380,780	\$ 4,732,234,130	\$ 4,723,885,714
Fuel	\$ 2,179,788,296	\$ 2,178,284,952	\$ 2,177,087,584	\$ 2,175,594,208
Other Op	\$ 2,335,830,900	\$ 2,334,705,422	\$ 2,337,440,544	\$ 2,336,141,584
Toll	\$ 104,087,516	\$ 107,126,108	\$ 104,039,788	\$ 106,509,752
Total	\$ 11,064,950,780	\$ 11,057,252,802	\$ 11,010,197,696	\$ 11,004,541,184

Table 4.7 Present value of benefits by assignment algorithm

	Cube	TAPAS	Cube - TAPAS	% Difference
Crash	\$ 374,370,701.40	\$ 358,853,323.97	\$ 15,517,377.43	4.14%
Time	\$ 268,353,533.55	\$ 255,742,456.67	\$ 12,611,076.88	4.70%
Fuel	\$ 32,117,694.19	\$ 32,001,579.49	\$ 116,114.70	0.36%
Other Op	\$ (19,685,723.78)	\$ (18,805,147.74)	\$ (880,576.04)	4.47%
Toll	\$ 588,221.95	\$ 3,930,895.68	\$ (3,342,673.74)	-568.27%
Total	\$ 655,744,844.78	\$ 631,724,281.98	\$ 24,020,562.80	3.66%

The differences in the two solutions are most striking with respect to Tolls, which are a relatively small component of overall costs and benefits.

4.7.3 Conclusions of the collaborator

The above results suggest the benefits from this project may be overstated by about four percent because the CUBE solutions were not as converged as the TAPAS solutions.

The computational effort required for computing the solutions with the CUBE software was relatively onerous from ODOT's point of view. While computational efficiency was not an objective of the present study, ODOT staff noted that they intend to increase the number of classes in the assignment in the future, which has substantial implications for computational effort.

The use of link-specific PCEs is another feature of ODOT's desired model that requires additional consideration and investigation.

5. Analyses of One-Class and Two-Class Flows on the Chicago Regional Network

5.1 Overview and objectives

At the outset of this project, the investigators had a clear understanding of the properties of their assignment method, TAPAS, but relatively little experience with its application to the types of large-scale road networks and origin-destination trip matrices used in practice. Moreover, we had little understanding of how software systems applied in practice would perform in identifying origin-destination (OD) and link flows over a selected network link. In order to explore these issues, we performed a number of select link and related analyses with a version of the Chicago regional road network coded by the Chicago Area Transportation Study (CATS) and extensively used during the 1990s. The CATS network has 1,790 zones, 12,982 nodes and 39,018 links. This network in turn was used by Bar-Gera and Boyce (2005, 2007) to create one-class and two-class trip matrices. The Truck trip matrix, provided by CATS, and has 445,185 car equivalent trips per hour. The Car trip matrix was created with a combined origin-destination, mode and road assignment model, given CATS's estimates of person trip origins and trip destination zone totals for the morning peak period for 1990, and has 984,717 car trips per hour. The one-class trip matrix, the sum of Car and Truck matrices, has 1,429,901 vehicles per hour.

Selected results from these studies are presented in this chapter. In all, four directional road links were chosen for select link analysis: North Avenue eastbound and westbound, an urban minor arterial two miles north of the Chicago CBD; Harlem Avenue northbound and southbound, a suburban minor arterial about ten miles west of the Chicago CBD. Three pairs of alternative segments (PAS), or sequences of directed links, were studied: two eastbound segments from the Kennedy Expressway ending at a zone near the lakefront, one of which uses North Avenue; two southbound segments ending at the north entrance to the Lake Shore Drive; two northbound segments beginning from the terminus of Illinois 53 in northern Cook County. In each case, the travel costs over the alternative segments are equal in a precisely converged assignment.

Of these several cases, the North Avenue eastbound and Harlem Avenue southbound select link analyses are presented here for the one-class and two-class cases. Then, analyses are presented for the one-class and two-class cases of the eastbound pair of alternative segments in the vicinity of North Avenue. The selected link, 6380 – 6389, is shown in Figure 5.1 below. The pair of alternative segments connects node 8032 to node 10345. The route over the selected link, 6380 – 6389, is segment 1; the alternative route is segment 2.

The OD flows found by four practitioner software systems are presented for the one-class case; corresponding link flows are presented for two systems. For the two-class case, the OD flows and corresponding link flows are presented for two systems. For the one-class cases, results for three types of assignment algorithms are compared with the results produced by TAPAS: link-based; route-based; and origin-based. For the two-class case, results for link-based and route-based algorithms are compared with results from TAPAS. For the four systems, the assignments were converged to a Relative Gap of 1E-04, as defined by each software system. For TAPAS, the results were converged to a more precise Relative Gap, equal to 2.6E-16 for the one-class case and 3.3E-16 for the two-class case. The TAPAS solution therefore provides a highly converged benchmark solution for both cases.



Figure 5.1 North Avenue select link and pair of alternative segments

Link-based assignments were performed using EMME 3.1.7, TransCAD 5.0 and CUBE 5.0. As the link-based results found with these three software systems are essentially the same, only results for EMME and CUBE are presented here. Route-based assignments were performed with VISUM 10 and a beta version of EMME 3.1.7. As the results found were quite similar, only the VISUM results are presented here. Results from an origin-based assignment performed with TransCAD 5, using the OUE option, complete the four sets of results, as shown in Table 5.1.

These results are presented to inform practitioners of the characteristics of TAPAS and the algorithms available from software developers. Interpretations and comparisons of the results are offered at the end of each subsection. For each set of results, plots are presented on linear and logarithmic scales. The linear plots show the relative size of the OD and link flows, but tend to obscure flows less than ten vehicle per hour. The logarithmic plots emphasize the small flows down to values of 1E-04 vehicles per hour, but tend to obscure large flows. We believe both plots are useful in interpreting the findings. Although very small flows have little effect on the numerical results, they could substantially increase the number and location of OD pairs identified in a select link analysis. Similar plots for link flows are also presented, together with maps showing the location and magnitude of the link flows.

Table 5.1 Overview of results presented in Chapter 5

Analysis and level		SLA - OD Flow		SLA - Link Flow		PAS - North Avenue	
Class		North eb	Harlem sb	North eb	Harlem sb	OD Flow	Link Flow
One	System applied	CUBE EMME TransCAD VISUM	CUBE EMME TransCAD VISUM	TAPAS EMME VISUM	TAPAS EMME VISUM	EMME VISUM	EMME VISUM
	Figure	5.2	5.3	5.4	5.5	5.6	5.7
Two	System applied	CUBE VISUM	CUBE VISUM	CUBE VISUM	CUBE VISUM	CUBE VISUM	CUBE VISUM
	Figure	5.8	5.9	5.10	5.11	5.12-5.13	5.14

5.2 One-class analyses

5.2.1 Select link analyses for North Avenue eastbound and Harlem Avenue southbound

A select link analysis (SLA) traces the flows passing over a selected link or segment (i.e., a sequence of links). Selected route flows may be traced by OD pair, which we refer to as the *OD-trace* of the selected link or segment; or they may be traced by other links in the network, which we refer to as the *link-trace* of the selected link or segment. To be more specific, for a given link or segment we identify all the routes that pass through the link or through the entire segment. When flows on the selected routes are aggregated by OD pair, they determine the OD-trace. When flows on the selected routes are aggregated by link they determine the link-trace.

Since route flows are not uniquely determined by the user-equilibrium (UE) conditions, various assignment methods may lead to different results. In the case of TAPAS, an assumption of proportionality is imposed, as explained in Chapter 2. For conventional assignment methods, route flows are undetermined, and required only to sum to the user-equilibrium link flows. The purpose of comparing results among several assignment methods for a selected link is to determine whether in a realistic setting the differences in the solutions have practical significance. In the following plots and maps, the selected flows are presented by their OD-trace and in some cases by their link-trace.

Figure 5.2 (a-d) shows the one-class SLA results for four practitioner software systems vs. TAPAS for the North Avenue eastbound OD-Trace. Each point in the first and third rows of plots represents the vehicle flow for one OD pair on a linear scale as found by a practitioner software (y-axis) and TAPAS (x-axis). The coordinates of each point indicate the vehicle flow per hour for that OD pair on routes that traverse North Avenue eastbound. The numbers of OD pairs using at least one route traversing North Avenue eastbound, as found by each software system, are shown on the axis labels. The total number of OD pairs found by the software or TAPAS is shown in the body of the plot. The Root Mean Square Error (RMSE) compares the two arrays. Each point in the second and fourth rows of plots represents vehicle flow for one OD pair on a \log_{10} scale. Values less than $1E-4$, including 0.0, are shown as $1E-4$. The numbers on the axes are the logs of the flows. For example, an axis value of 0.0 indicates a flow equal to 1.0.

Figure 5.3 (a-d) shows the results for four practitioner software systems for the Harlem Avenue southbound OD-Trace with the same arrangement of the plots.

Figure 5.4 (a, b) shows the North Avenue eastbound Link Flow (column 1), and Figure 5.5 (a-b) shows the Harlem Avenue southbound Link Flow (column 2), for two software systems. The coordinates of each point show the aggregate vehicle flow per hour on all routes that traverse both that link and North Avenue eastbound or Harlem Avenue southbound, as found by EMME, VISUM and TAPAS. The numbers of links in the trace found by each software system are shown on the axis labels. The total number of links in at least one of the two traces, as found by the two software systems, is shown above the plot. RMSE compares the two arrays. Connectors between zone centroids and the network are excluded from these plots. Figures 5.4 (c-e) and 5.5 (c-e) show corresponding maps of the location and relative size of link flows.

Figure 5.2 North Avenue eastbound OD flow

a) Link-based assignment: CUBE

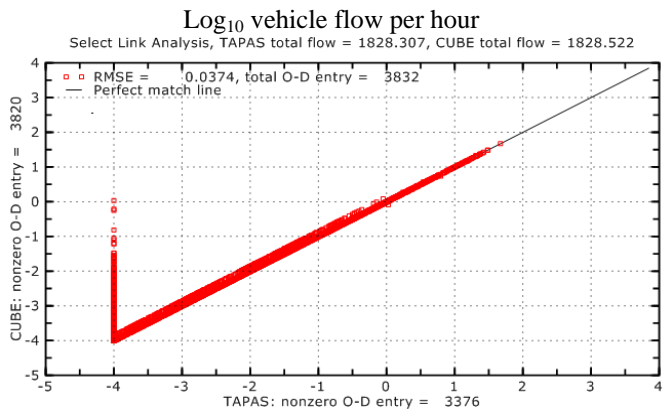
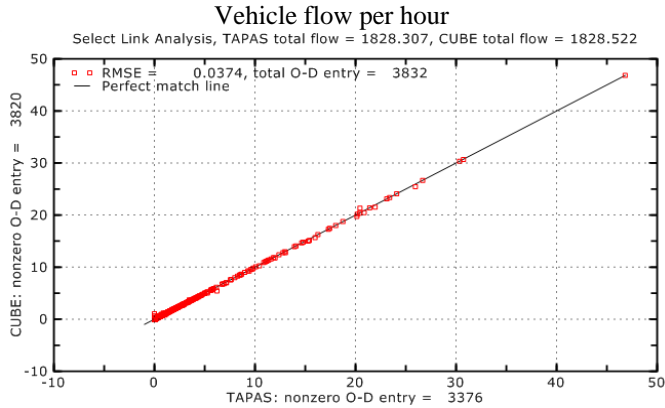
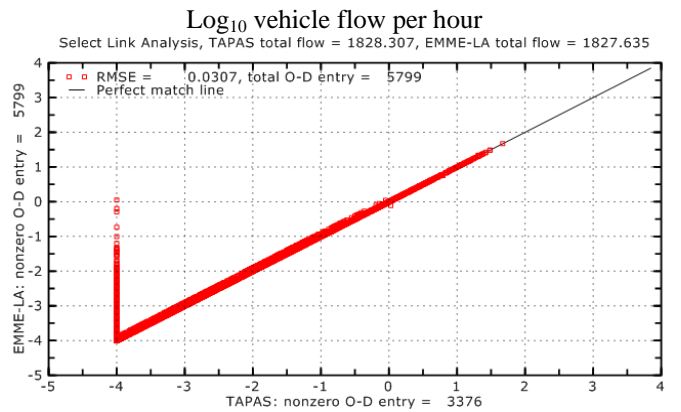
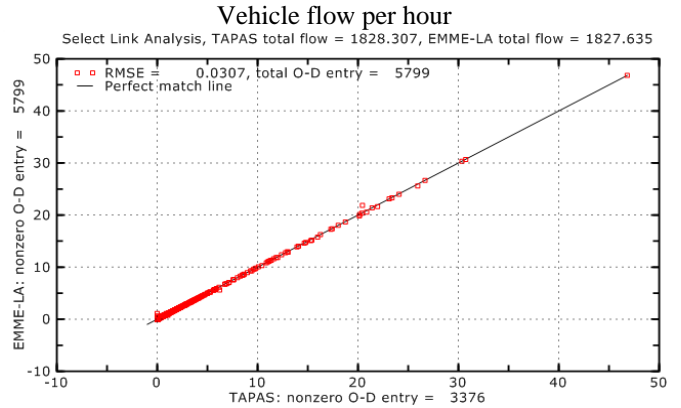
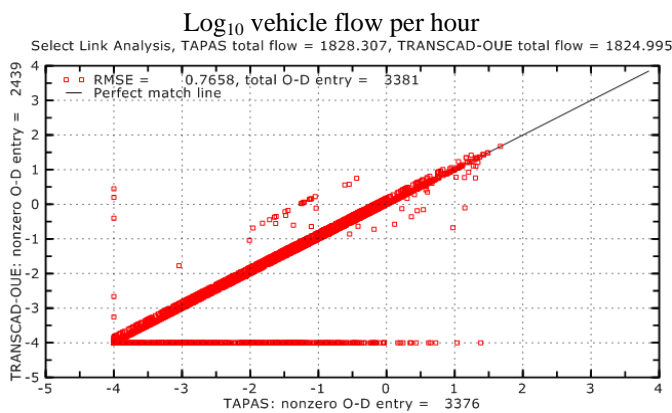
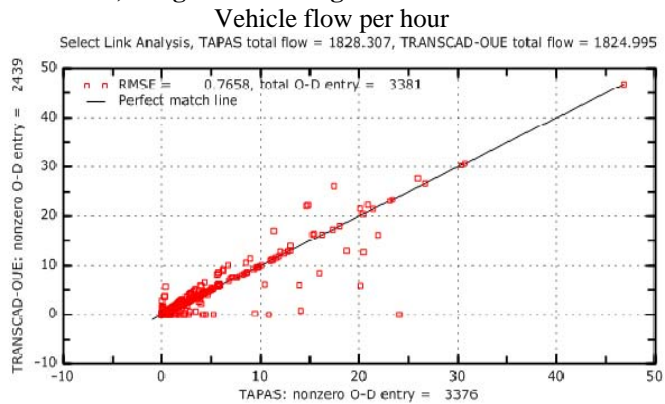


Figure 5.2 North Avenue eastbound OD flow

b) Link-based Assignment: EMME



c) Origin-based assignment: TransCAD



d) Route-based assignment: VISUM

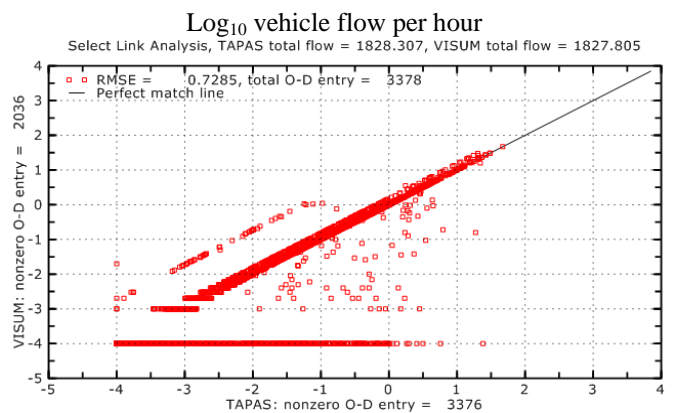
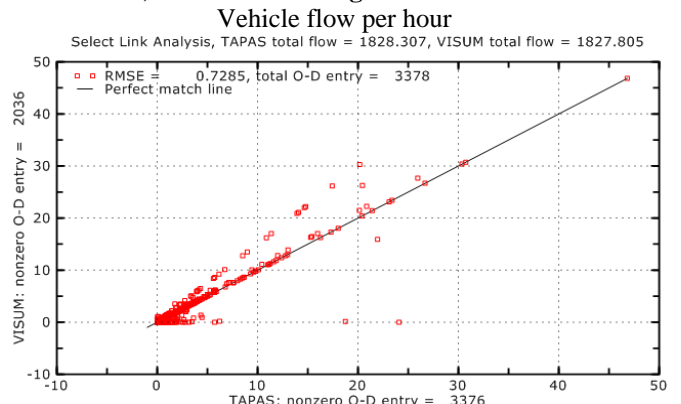
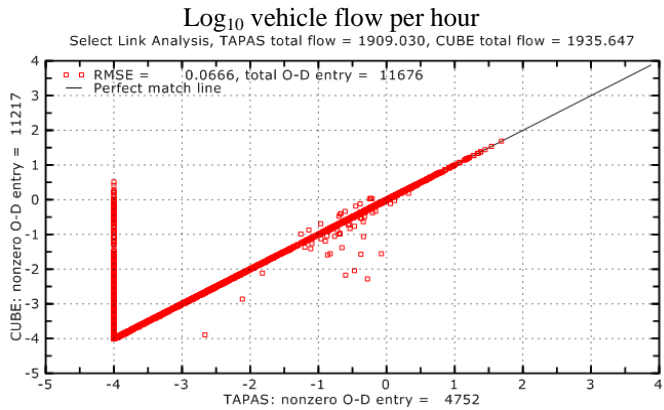
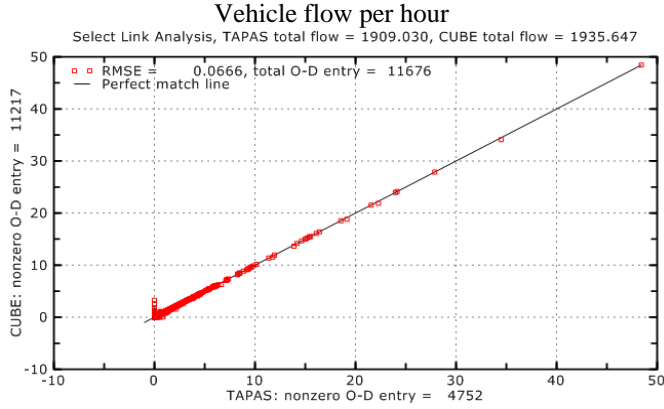


Figure 5.3 Harlem Avenue southbound OD flow

a) Link-based Assignment: CUBE



c) Origin-based assignment: TransCAD

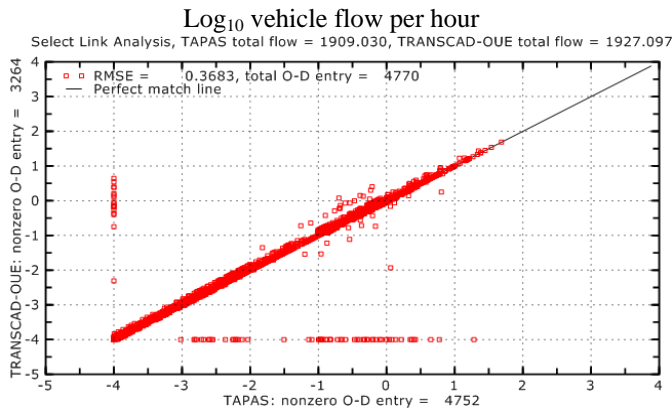
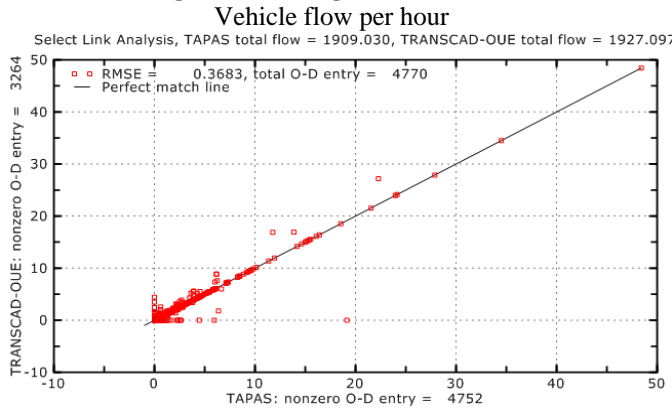
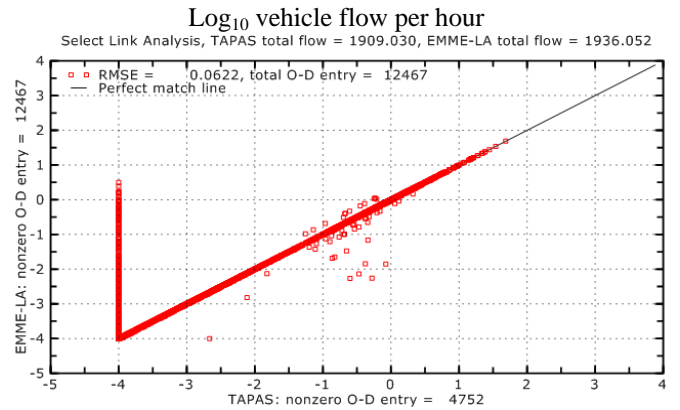
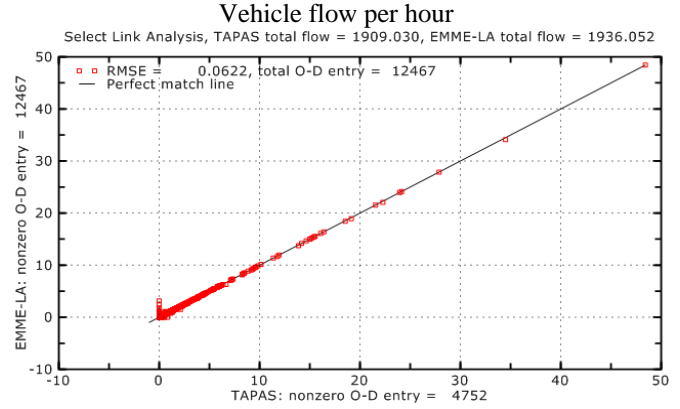


Figure 5.3 Harlem Avenue southbound OD flow

b) Link-based Assignment: EMME



d) Route-based assignment: VISUM

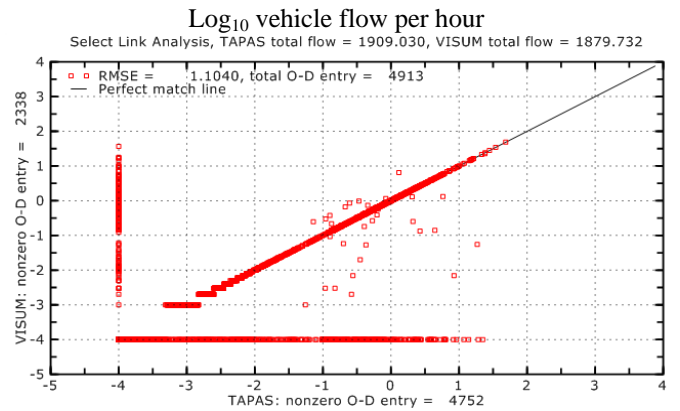
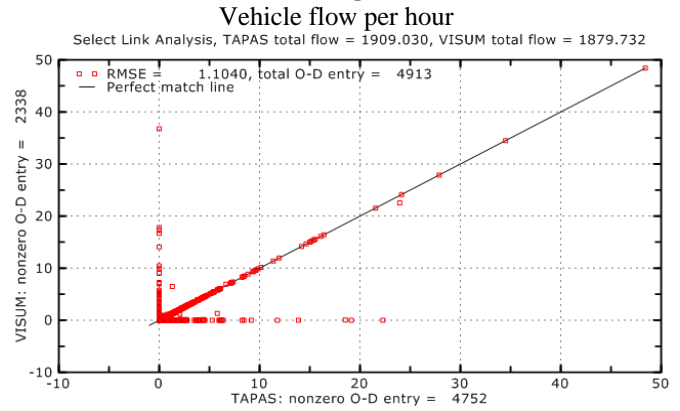


Figure 5.4 North Avenue eastbound link flow

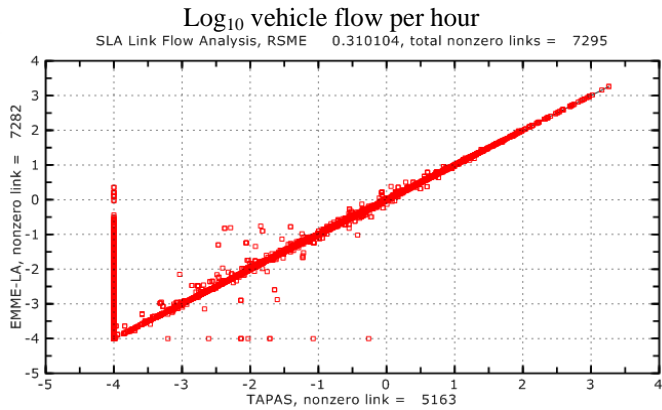
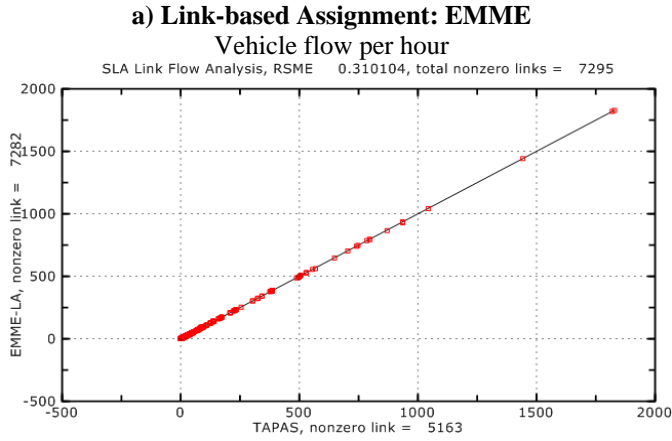


Figure 5.5 Harlem Avenue southbound link flow

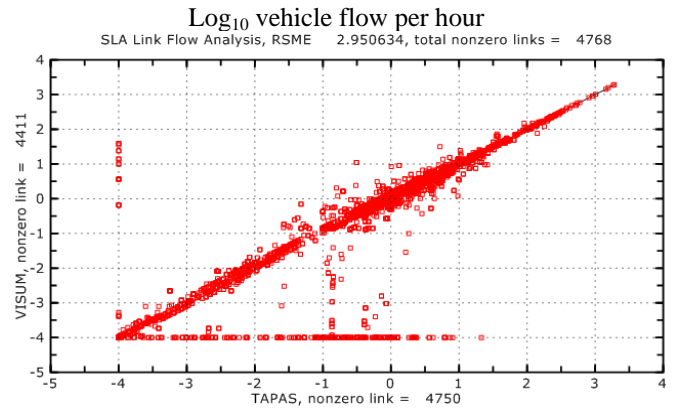
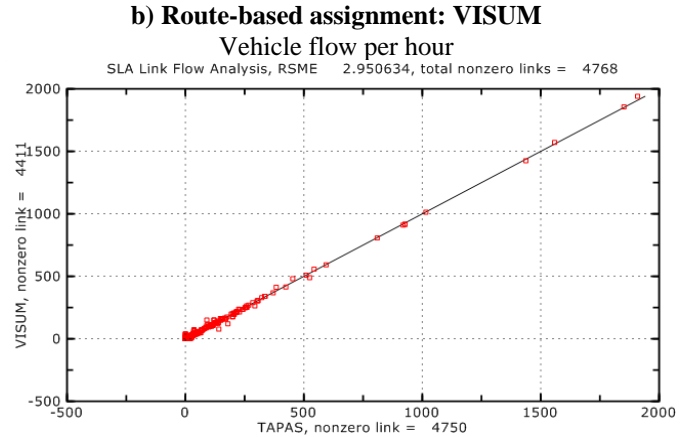
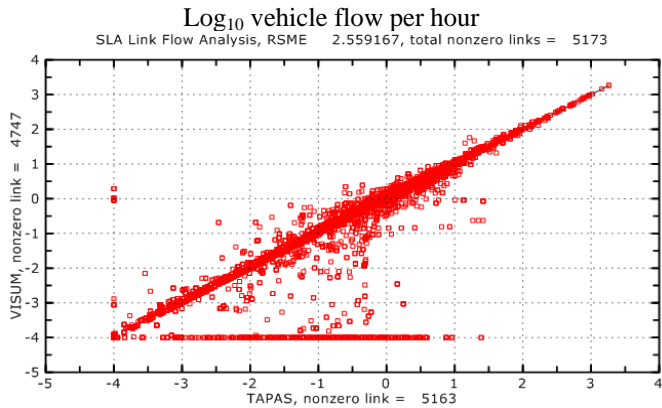
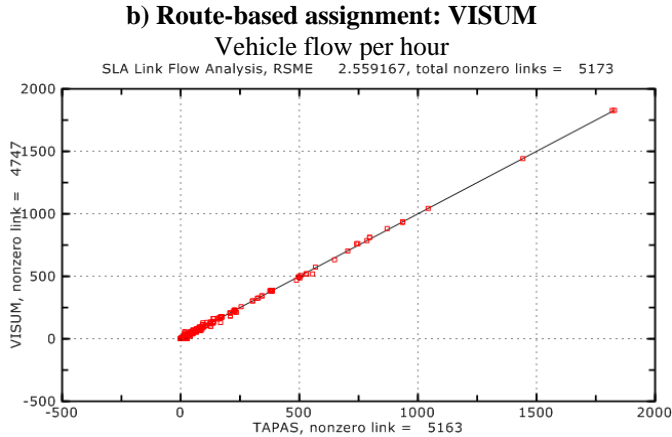
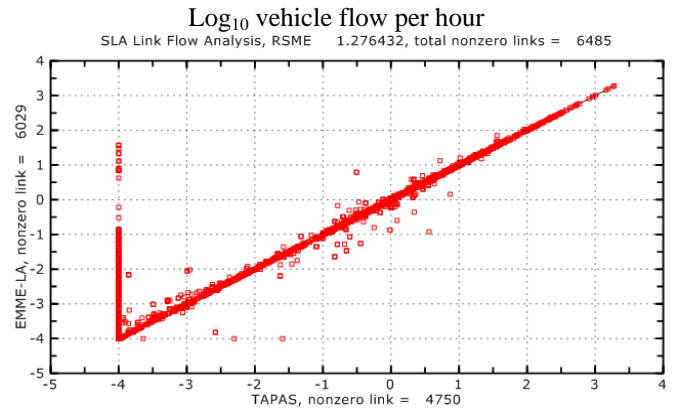
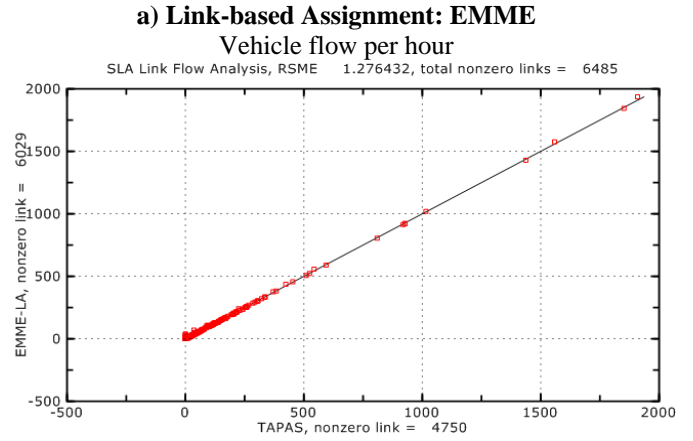
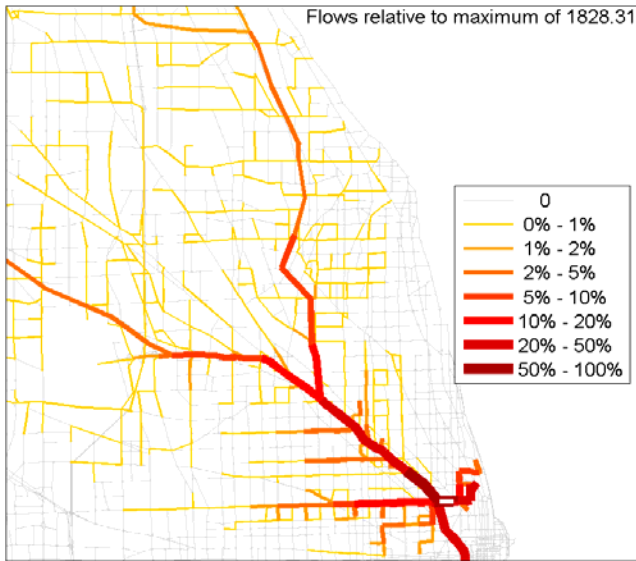
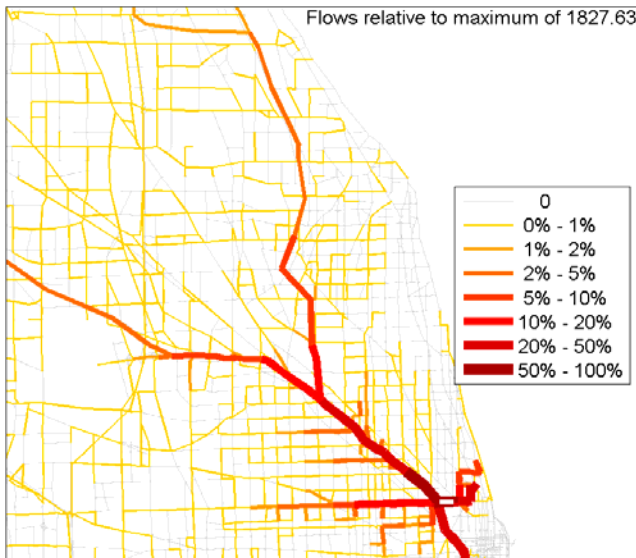


Figure 5.4 North Avenue eastbound link map

c) TAPAS



d) Link-based assignment: EMME



e) Route-based assignment: VISUM

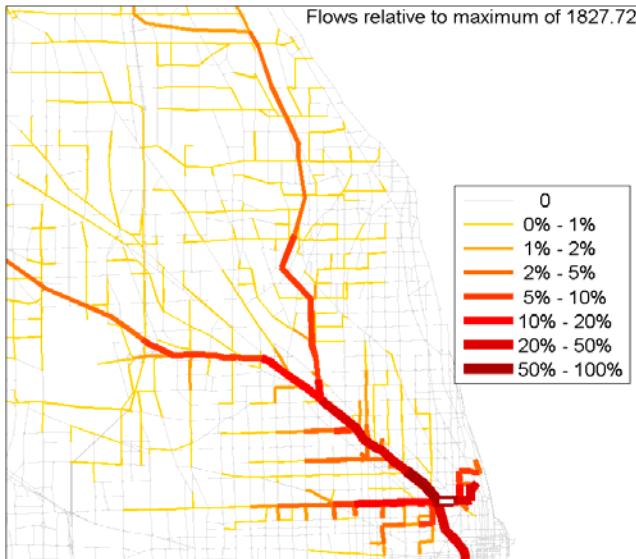
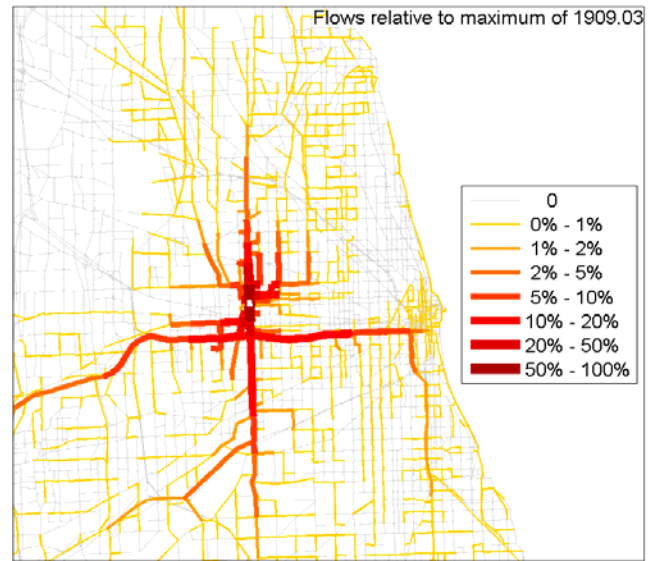
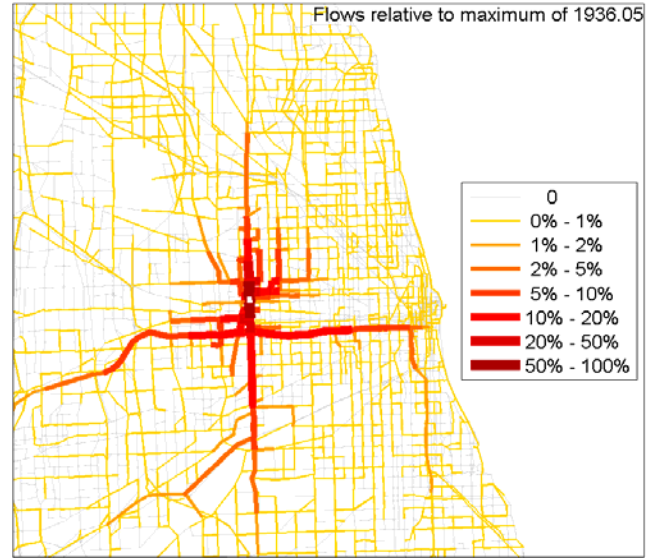


Figure 5.5 Harlem Avenue southbound link map

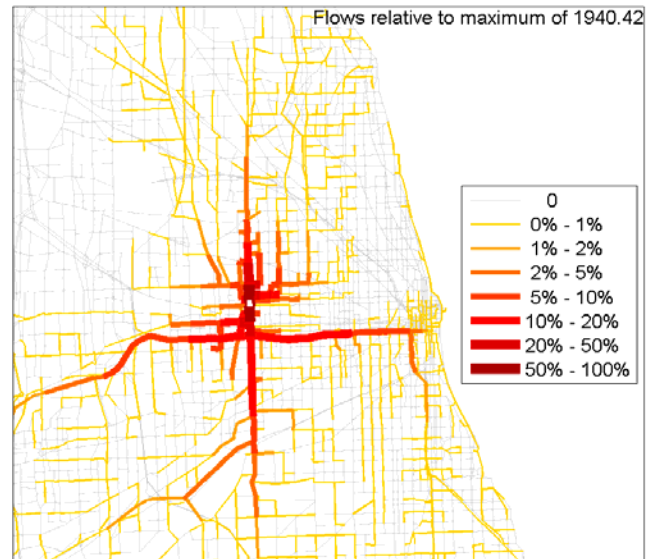
c) TAPAS



d) Link-based assignment: EMME



e) Route-based assignment: VISUM



5.2.2 Discussion of one-class select link analyses

We offer several observations about the OD-traces and link-traces shown in Figures 5.2 to 5.5:

1. The link-based assignment methods identify OD flows that are quite similar to those found by TAPAS. This observation is illustrated here for CUBE and EMME-LA (linear approximation), and also was observed for TransCAD-FW. The similarity between the link-based methods and TAPAS was somewhat unexpected; however, it may not occur in more congested networks or in networks coded differently.
2. The primary difference between the link-based assignments and TAPAS is the number of OD pairs identified. The link-based methods often find far more OD pairs than the highly converged TAPAS solution. This difference is characteristic of link-based methods in which an OD pair included in the solution at an early iteration is never completely removed. This effect is easily observed in the \log_{10} plots and in the maps of link flows.
3. The origin-based and route-based solutions are somewhat different from the TAPAS solution. Such differences were also found in other examples considered. Unlike the TAPAS solution in which the assumption of proportionality is enforced for every pair of alternative segments in the network, the UE link flow is the sole objective for these two solution methods.
4. The origin-based and route-based solutions tend to find far fewer OD pairs than TAPAS. This effect is also easily seen in the \log_{10} plots and in the maps of the link flows.

If the number and magnitude of route flows by OD pair and the magnitude and location of link flows are desired for policy analyses, then we may conclude that none of the practitioner methods adequately determines the desired result.

5.2.3 One-class analysis of a pair of alternative segments for North Avenue

Each pair of alternative segments is composed of two segments, which have precisely the same travel time in the TAPAS solution. One way to choose one route flow solution from all possible UE route flow solutions is to impose an assumption of proportionality. According to this assumption, the proportion of flow on each of the two segments is the same for all OD pairs using these segments.

Figure 5.6 (a-b) shows the PAS results for EMME and VISUM vs. TAPAS for a segment using North Avenue and another segment to the north, as was shown in Figure 5.1. Each point shown in the first row of plots represents one OD pair on a linear scale. The horizontal coordinate of each point indicates the vehicle flow per hour for the OD pair on routes that traverse segment 1; the vertical coordinate indicates the equivalent value for segment 2. Blue triangles show the results found by EMME (left) or VISUM (right) and red squares show the results found by TAPAS. The total flow on all the routes that traverse each segment is shown in the body of the plot for each software system. In a solution that satisfies the assumption of proportionality, all

the points should lie on a straight line through the origin (0, 0). The same information is shown in the second row in a log-log plot, which emphasizes the role of small flows.

The third row of plots provides distribution information concerning each OD pair: the vertical coordinate shows the log of the ratio of (a) vehicle flow on routes for the ODs that traverse segment 1 relative to (b) the vehicle flow on routes for the ODs that traverse segment 2. Blue triangles show the results as found by EMME or VISUM, and red squares show the results as found by TAPAS. If the OD flow is zero on one segment, the log of the ratio is shown as +3 or -3. If the assumption of proportionality is satisfied, then the ratio and the log of the ratio should be equal for all OD pairs; i.e. the plot should show a horizontal line. The total number of OD pairs with non-zero flow on one or both of the segments is shown in the body of the plot. The assumption of proportionality stated above implies that the set of OD pairs that use at least one of the two segments should be identical to the set of OD pairs that use both segments, so the two numbers should be equal.

Figure 5.7 (a-e) shows the North Avenue PAS link flows and maps for TAPAS, EMME and VISUM. Each point shown in top half of Figure 5.7 (a-b) represents one link on a linear scale for EMME (left) and VISUM (right); as before, zone connectors are excluded. The coordinates of each point indicate the vehicle flow per hour on routes traversing both the link and one of the two segments as found by EMME or VISUM vs. TAPAS. Each point shown in lower half of Figure 5.7 (a-b) represents one link on a \log_{10} scale. The coordinates of each point indicate the vehicle flow per hour on routes traversing both the link and one of the two segments as found by EMME (left) or VISUM (right) vs. TAPAS. The corresponding maps of link flows are shown as Figure 5.7 (c-e).

5.2.4 Discussion of one-class analysis of a pair of alternative segments

The link-based assignment, represented here by EMME-LA, corresponds very closely to the result found by TAPAS, both in terms of total OD flows on the two segments and the proportion of flows for each OD pair. This result is also observed in the log plot for very small flows and in the distribution of the log of the ratios of the segment flows. As with the select link analyses, the results for the route-based analysis are quite different. Although the total flows by segment are solved quite precisely, the segment flows are mainly placed on either one segment or the other. This dichotomy of flows can be seen clearly in the plot of the distribution of the logs of the ratios of segment flows. The link-based assignment exhibits errors for very small flows, from 0 to 0.1, whereas the route-based assignment shows errors for the larger flows, from 0.001 up to 30.

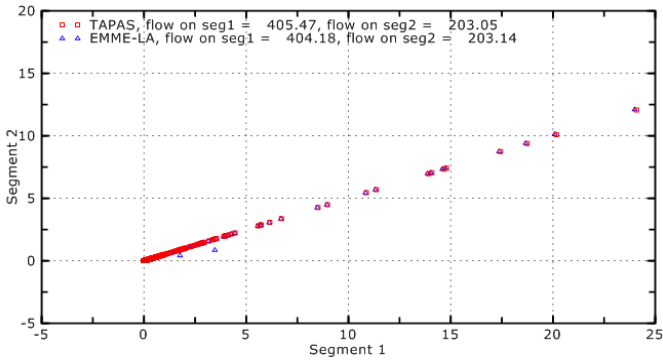
For the link-based method, the assigned link flows on segments 1 and 2 correspond very closely to those found by TAPAS. Link flows less than one vehicle per hour correspond less well to the TAPAS results. Moreover, the number of links with positive flow is somewhat higher for EMME than for TAPAS: 2,989 vs. 2,575. For the route-based method, the link flows correspond somewhat well to the TAPAS results for the larger flows. The number of links with flows given by VISUM is somewhat smaller than by TAPAS, 2,005 vs. 2,575. These omitted links are represented by the horizontal bar at the bottom of the log plots. Differences in the number of links with flows in the EMME and VISUM solutions are also quite evident in the maps.

Figure 5.6 North Avenue PAS OD flow

a) Link-based assignment: EMME

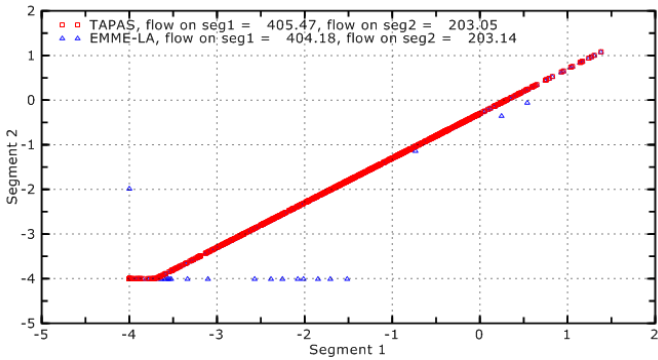
Vehicle flow per hour

Paired Segments Analysis



Log₁₀ vehicle flow per hour

Paired Segments Analysis



Distribution of log ratios

Paired Segments Analysis, O-D pairs used = 723

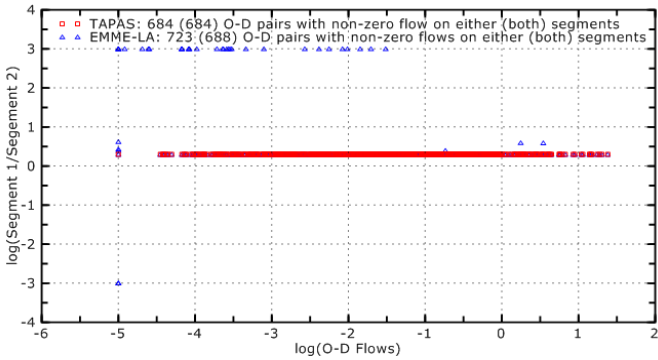
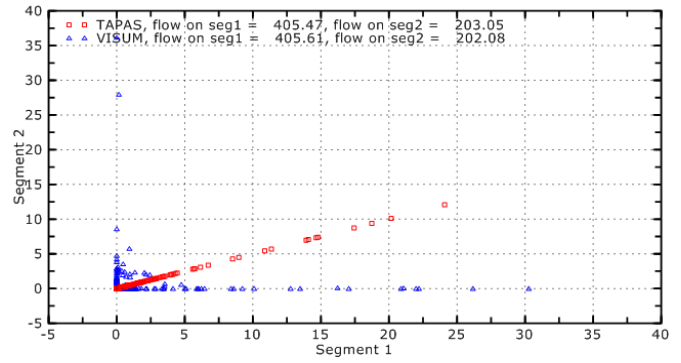


Figure 5.6 North Avenue PAS OD flow

b) Route-based assignment: VISUM

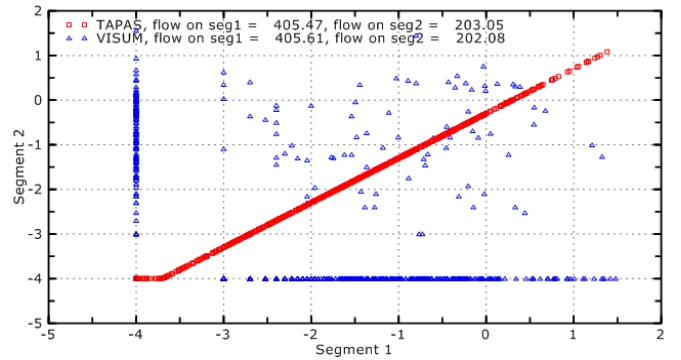
Vehicle flow per hour

Paired Segments Analysis



Log₁₀ vehicle flow per hour

Paired Segments Analysis



Distribution of log ratios

Paired Segments Analysis, O-D pairs used = 684

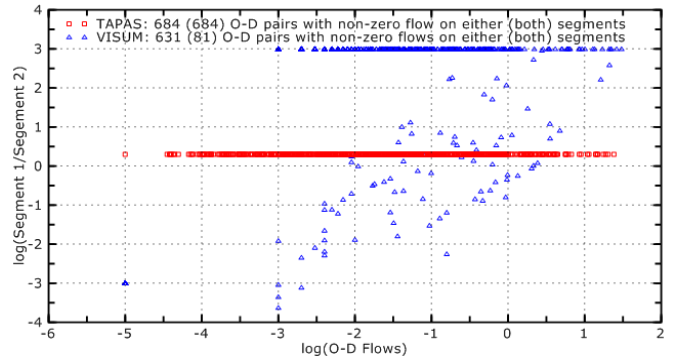
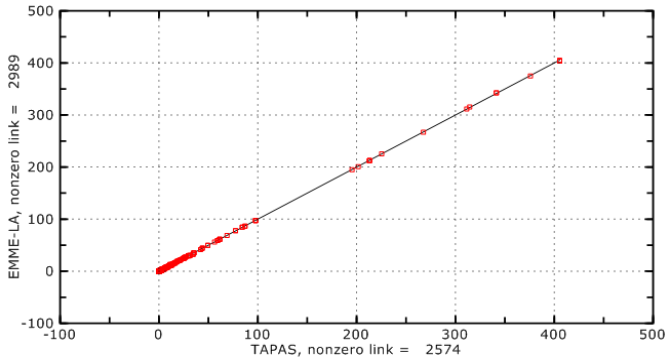


Figure 5.7 North Avenue PAS link flow

a) Link-based assignment: EMME

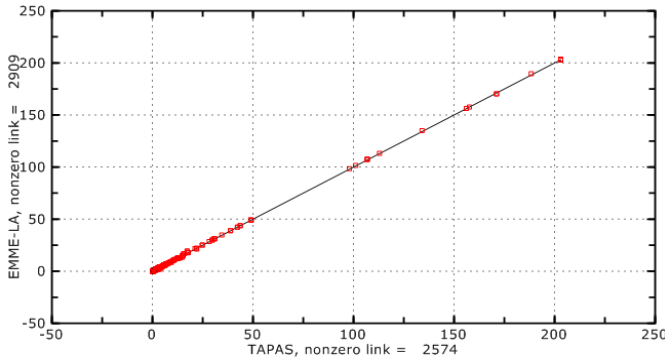
Vehicle flow per hour: segment 1

SLA Link Flow Analysis, RSME 0.154897, total nonzero links = 2998



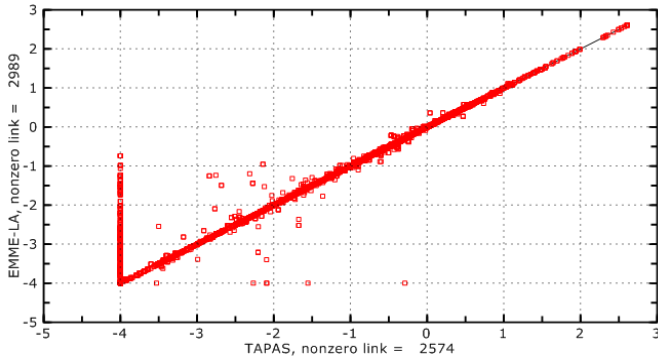
Vehicle flow per hour: segment 2

SLA Link Flow Analysis, RSME 0.141552, total nonzero links = 2922



Log₁₀ vehicle flow per hour: segment 1

SLA Link Flow Analysis, RSME 0.154897, total nonzero links = 2998



Log₁₀ vehicle flow per hour: segment 2

SLA Link Flow Analysis, RSME 0.141552, total nonzero links = 2922

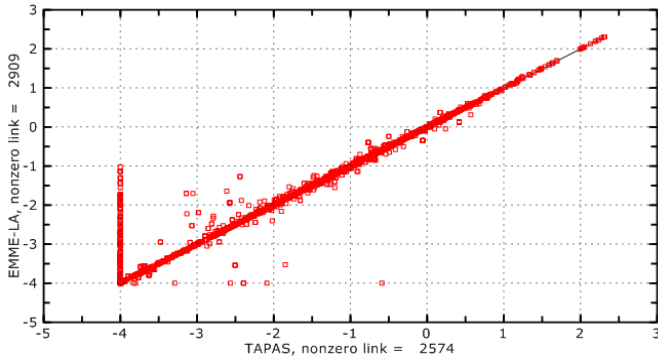
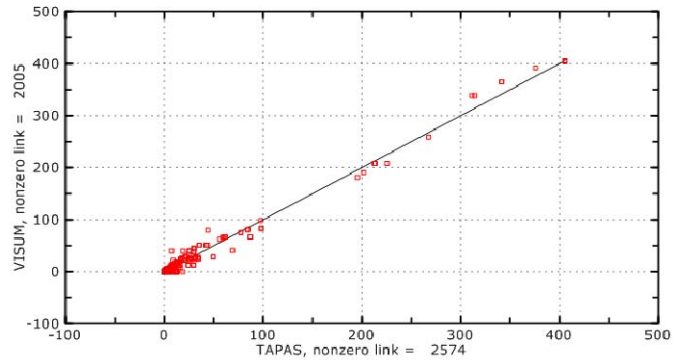


Figure 5.7 North Avenue PAS link flow

b) Route-based assignment: VISUM

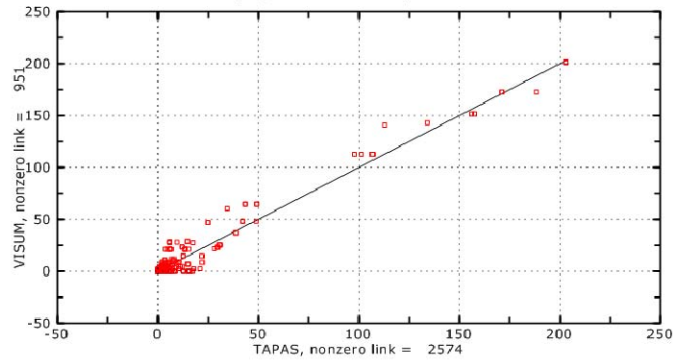
Vehicle flow per hour: segment 1

SLA Link Flow Analysis, RSME 2.632941, total nonzero links = 2574



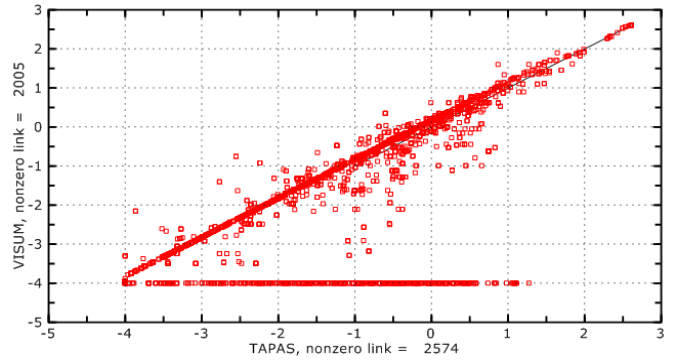
Vehicle flow per hour: segment 2

SLA Link Flow Analysis, RSME 2.506372, total nonzero links = 2574



Log₁₀ vehicle flow per hour: segment 1

SLA Link Flow Analysis, RSME 2.632941, total nonzero links = 2574



Log₁₀ vehicle flow per hour: segment 2

SLA Link Flow Analysis, RSME 2.506372, total nonzero links = 2574

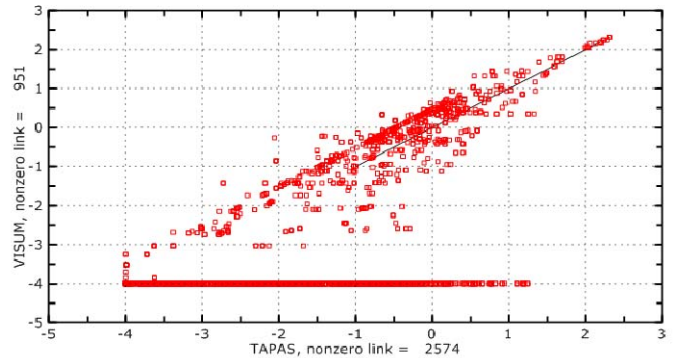


Figure 5.7 North Avenue PAS link map

c) Segment 1: TAPAS

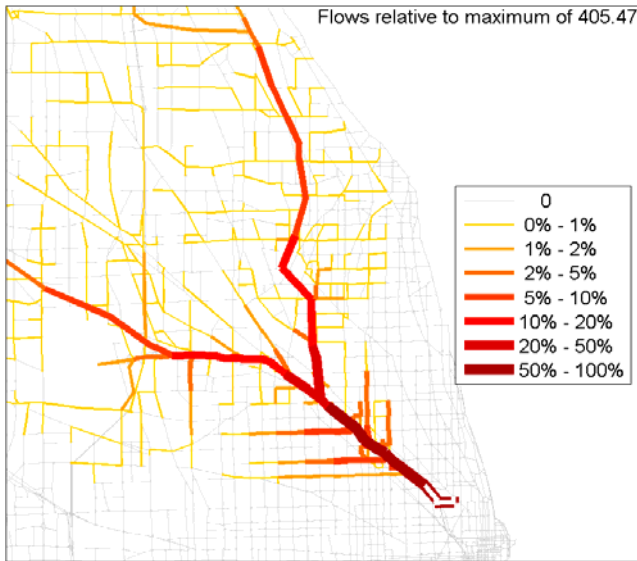
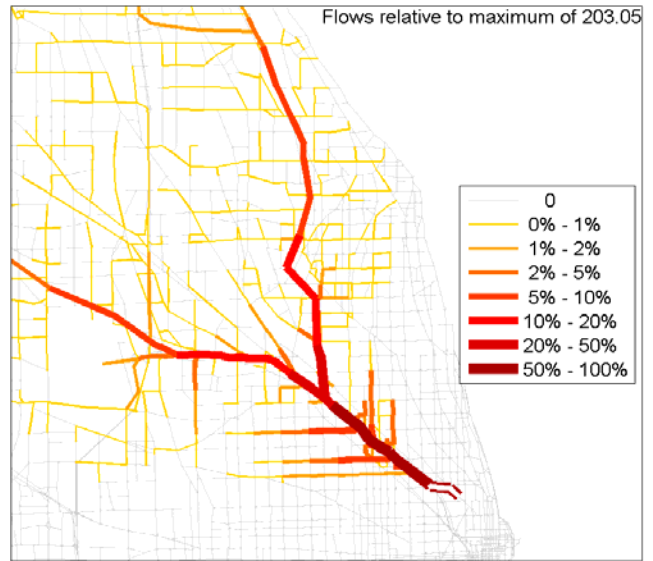
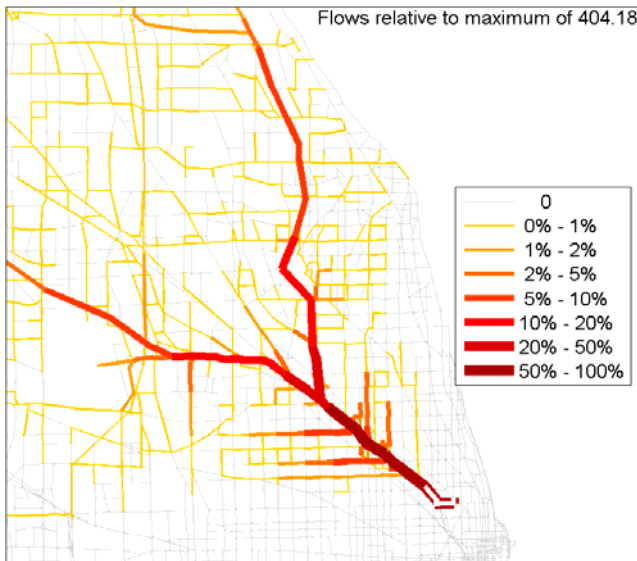


Figure 5.7 North Avenue PAS link map

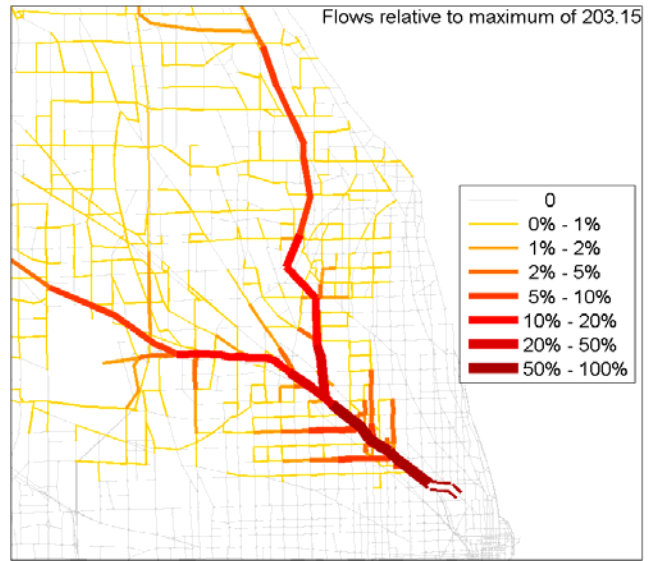
c) Segment 2: TAPAS



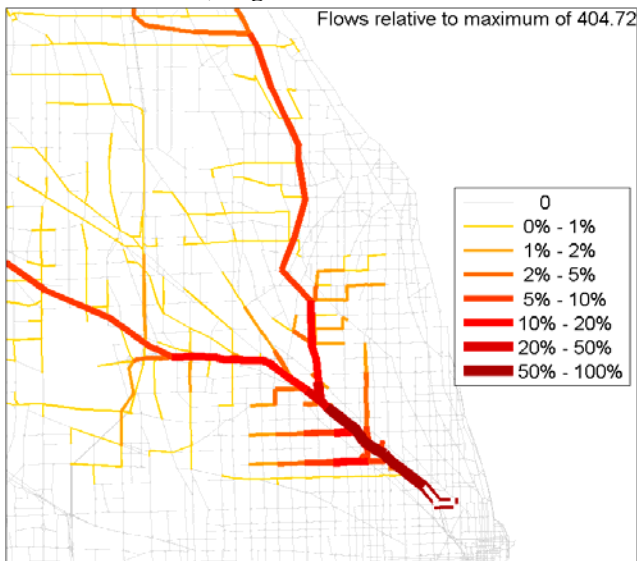
d) Segment 1: EMME



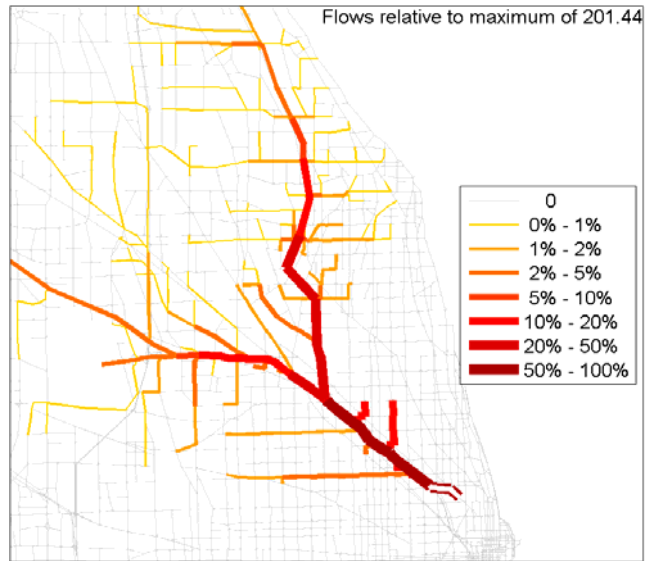
d) Segment 2: EMME



e) Segment 1: VISUM



e) Segment 2: VISUM



5.3 Two-class analyses

In this section, comparisons of two-class user equilibrium traffic assignments are presented for two trip matrices prepared using CUBE, VISUM and TAPAS. In this application, two classes of trips are assigned, cars and trucks, expressed as passenger car equivalents. The cost functions for the two classes are identical, namely travel time. The road network for the two classes is largely the same; however, certain facilities are not available for use by trucks: Lake Shore Drive, Kennedy Expressway express lanes and Dan Ryan Expressway express lanes. This section presents select link analyses for North Avenue eastbound and Harlem Avenue southbound and an analysis of a pair of alternative segments for North Avenue.

5.3.1 Select link analyses for North Avenue eastbound and Harlem Avenue southbound

Select link analysis for two-class assignment proceeds in the same way as for one class, except that there is a plot for each class. Results are shown in Figs. 5.8 – 5.11 for North Avenue eastbound and Harlem Avenue southbound for CUBE and VISUM vs. TAPAS. The layout of the plots is similar to the one-class case.

5.3.2 Discussion of two-class select link analyses

The OD flow results for two-class select link analysis are rather similar to the one-class analysis: the link-based method performs somewhat better than the route-based method; the link-based method substantially overstates the number of OD pairs using the selected link, whereas the route-based method understates the number using that link. The solutions for cars and trucks appear to be about equal in quality for this rather simple problem.

The solutions for the link-based and route-based methods for link flows are also rather similar to the OD flow results: the link-based method appears to provide a somewhat better prediction, but overstates the number of links with flow; see the RMSE for an overall comparison of results. The maps in Figures 5.10 and 5.11, if examined carefully, also illustrate the differences in the solution.

5.3.3 Two-class analysis of a pair of alternative segments for North Avenue

The pair of alternative segments analysis for two-class assignment also proceeds in the same way as the one-class analysis, with a plot for each class. Results are shown for the North Avenue PAS for CUBE and VISUM in Figures 5.12 – 5.15. The layout of the plots is similar to the one-class case.

Figure 5.8 North Avenue eastbound OD flow

a) Link-based assignment: CUBE

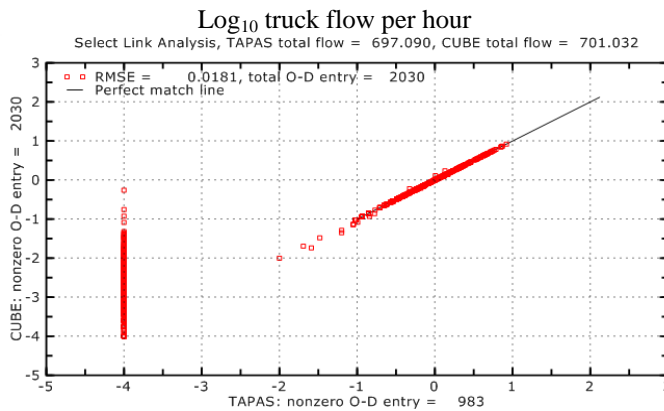
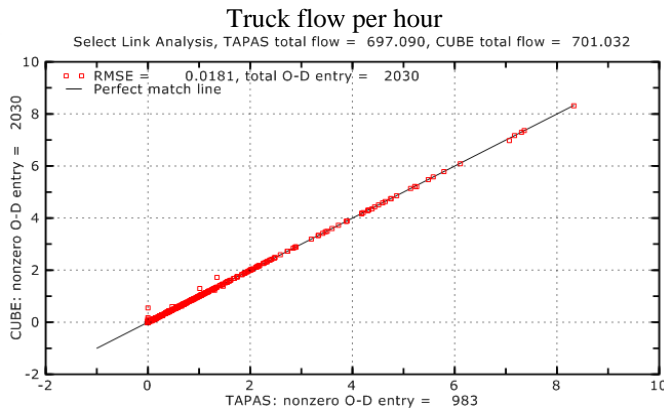
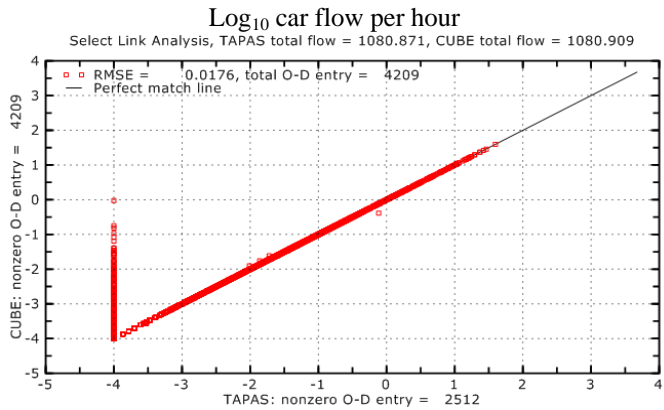
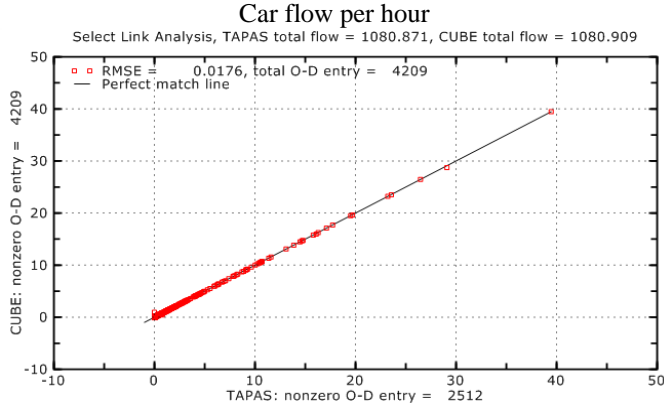


Figure 5.8 North Avenue eastbound OD flow

b) Route-based assignment: VISUM

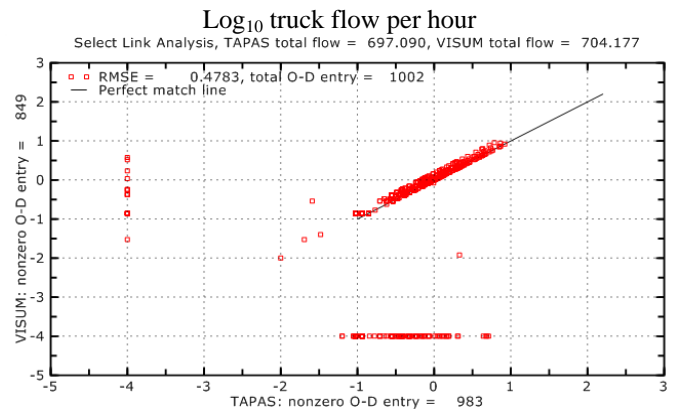
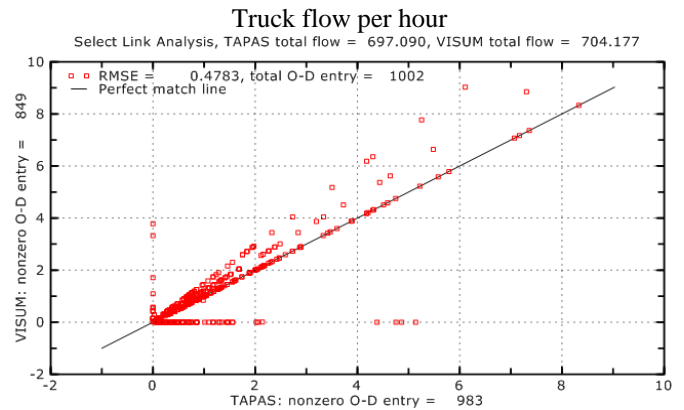
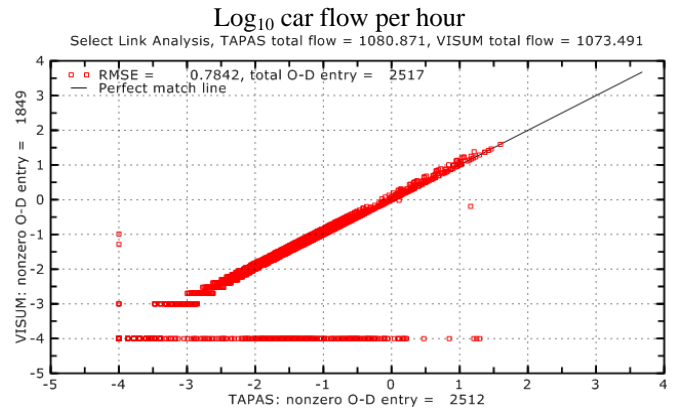
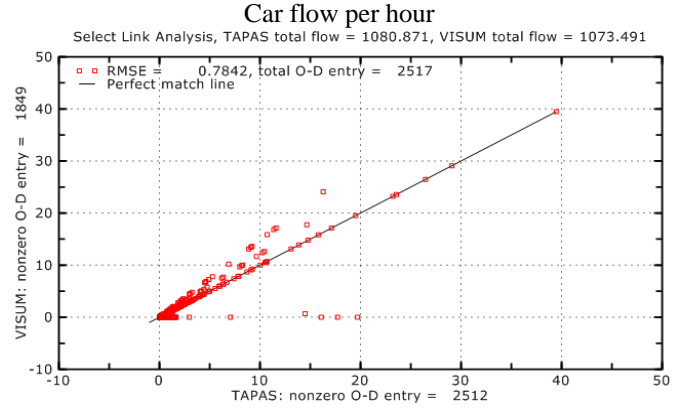


Figure 5.9 Harlem Avenue southbound OD flow

a) Link-based assignment: CUBE

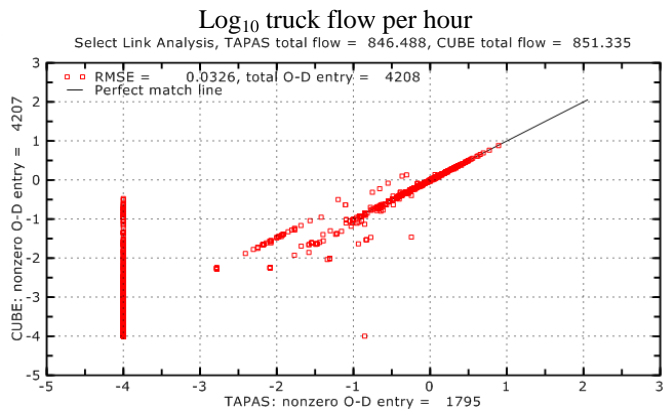
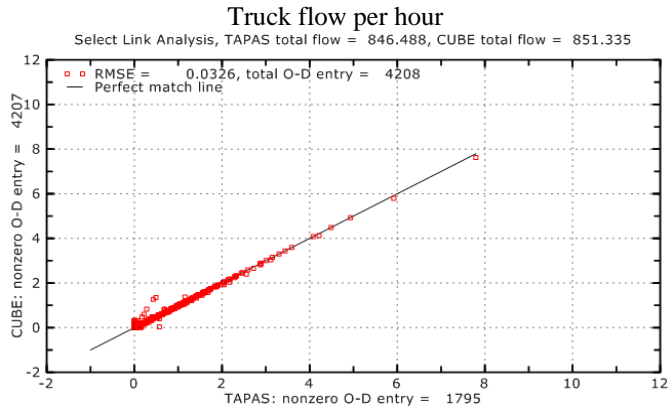
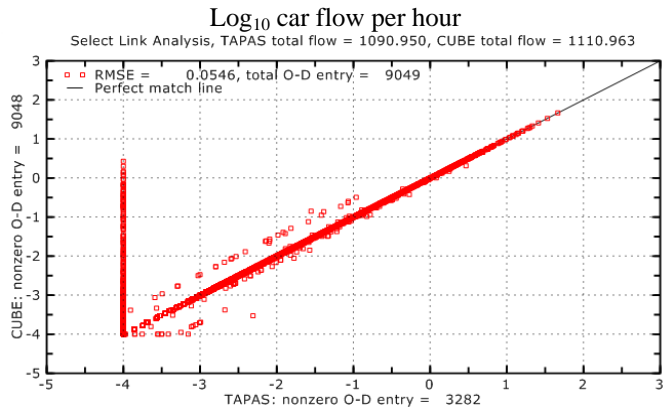
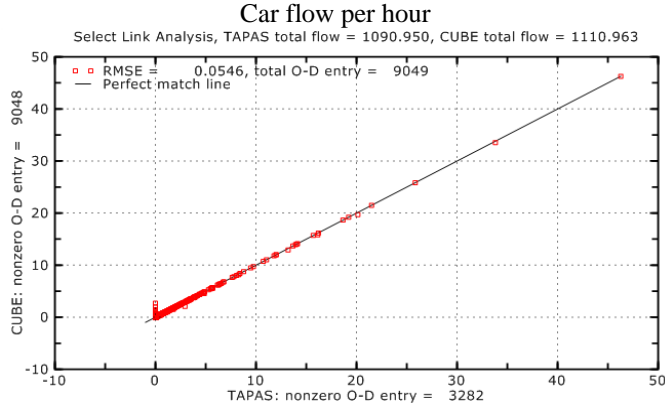


Figure 5.9 Harlem Avenue southbound OD flow

b) Route-based assignment: VISUM

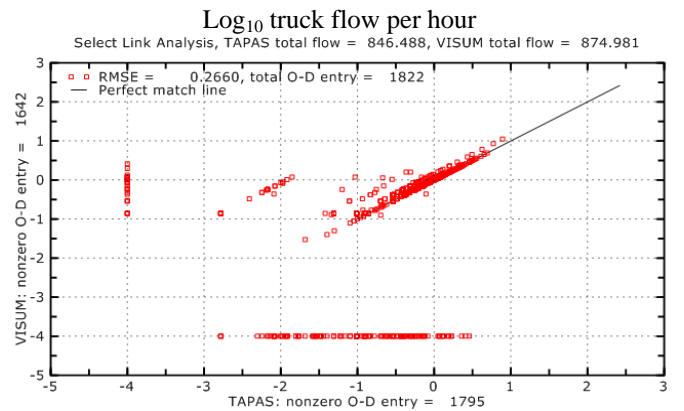
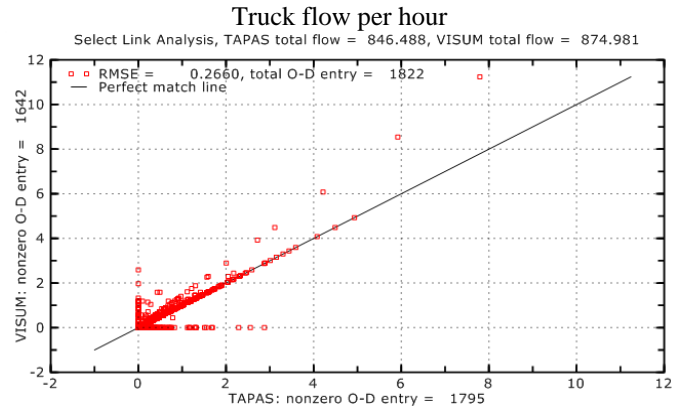
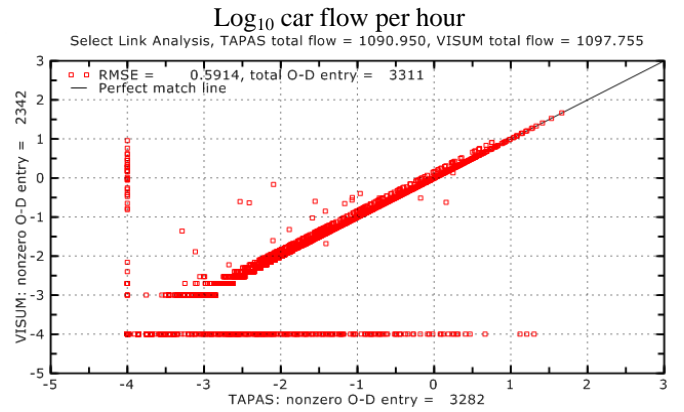
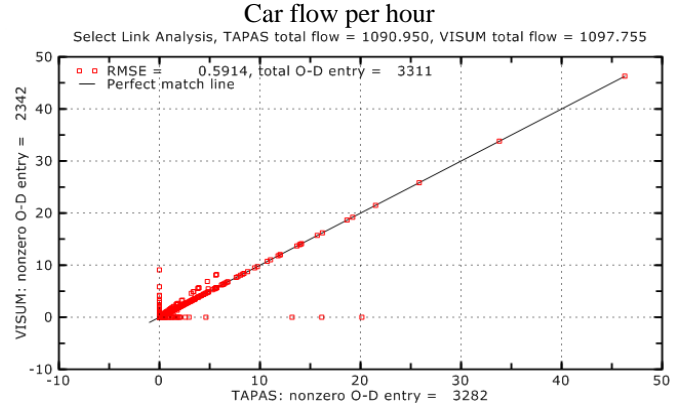


Figure 5.10 North Avenue eastbound link flow

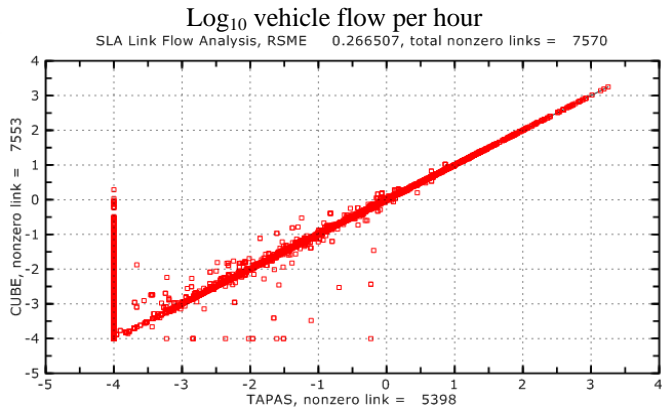
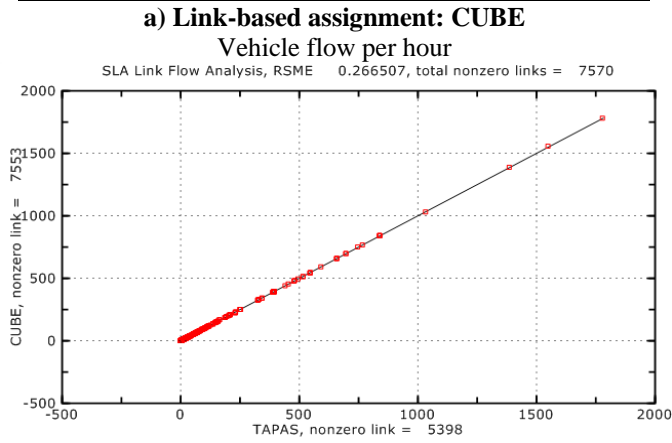


Figure 5.11 Harlem Avenue southbound link flow

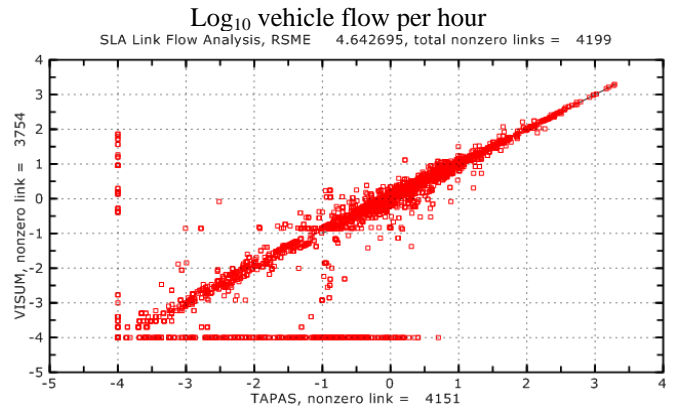
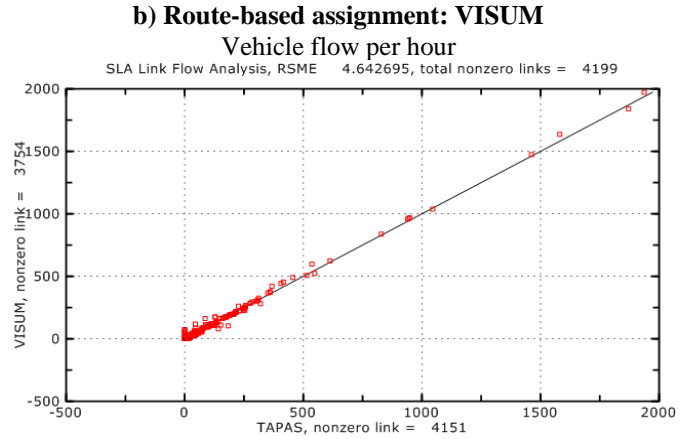
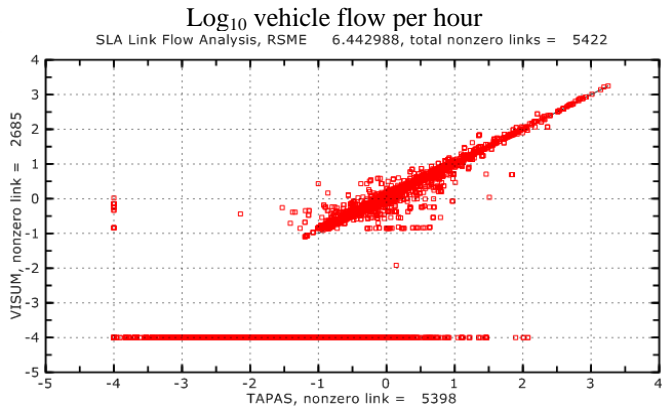
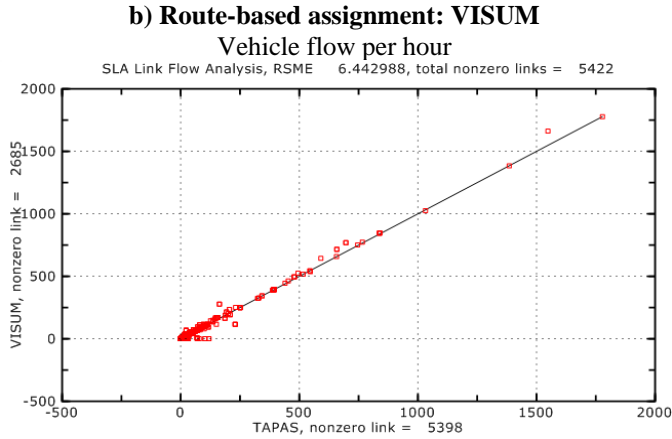
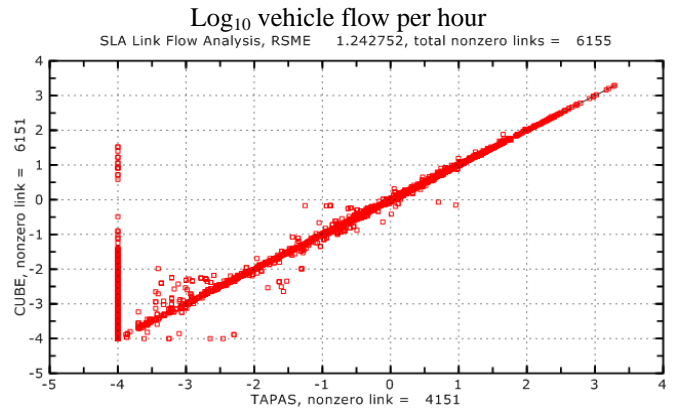
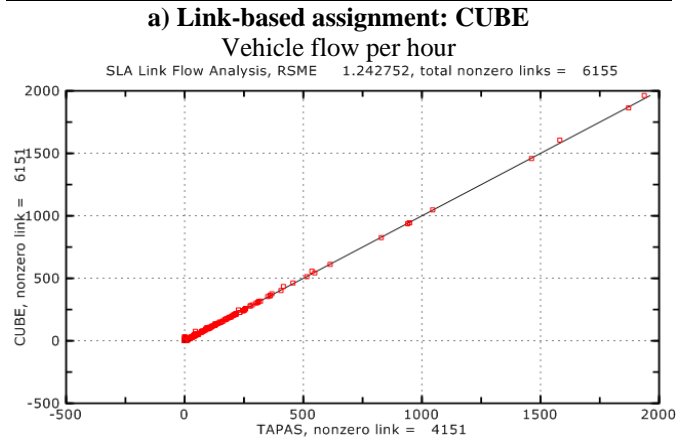
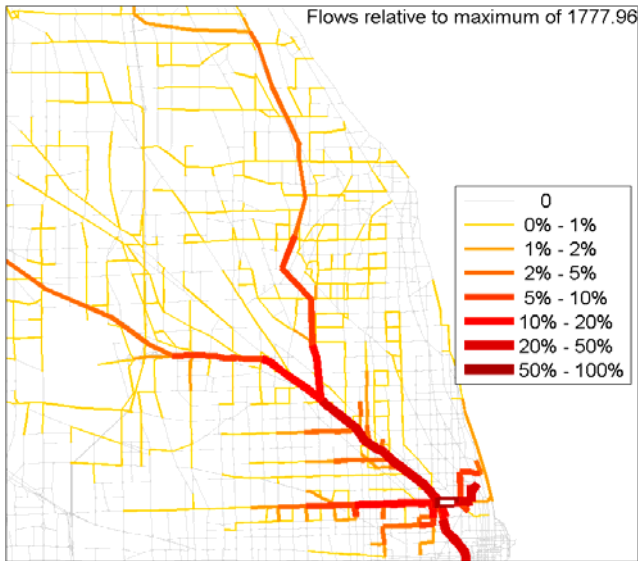
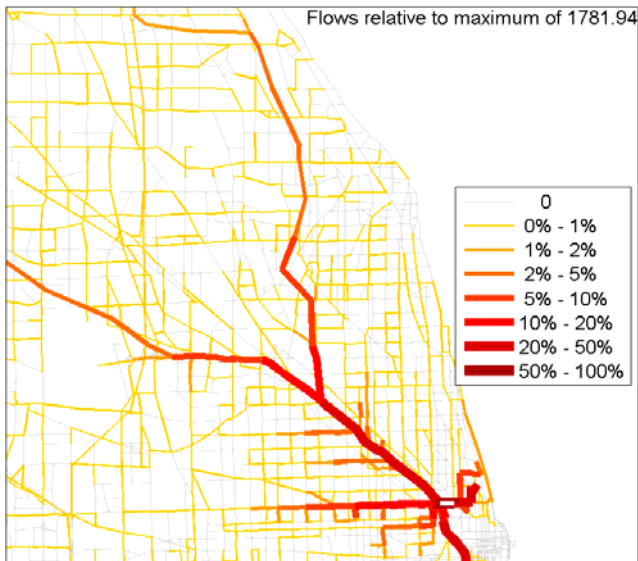


Figure 5.10 North Avenue eastbound link map

c) TAPAS



d) Link-based assignment: CUBE



e) Route-based assignment: VISUM

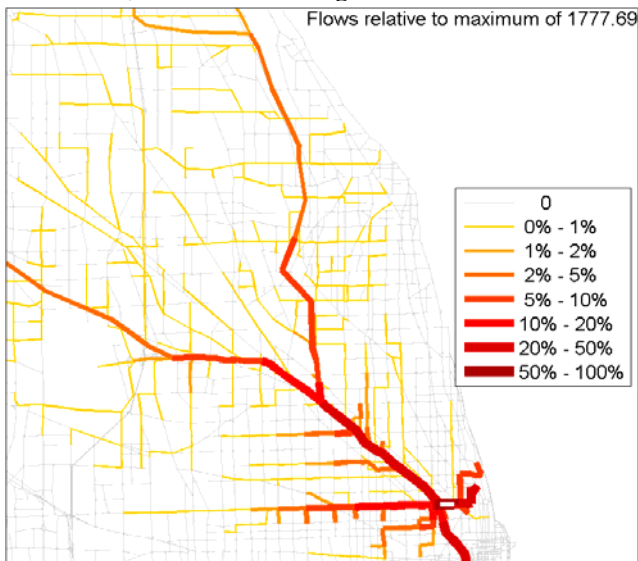
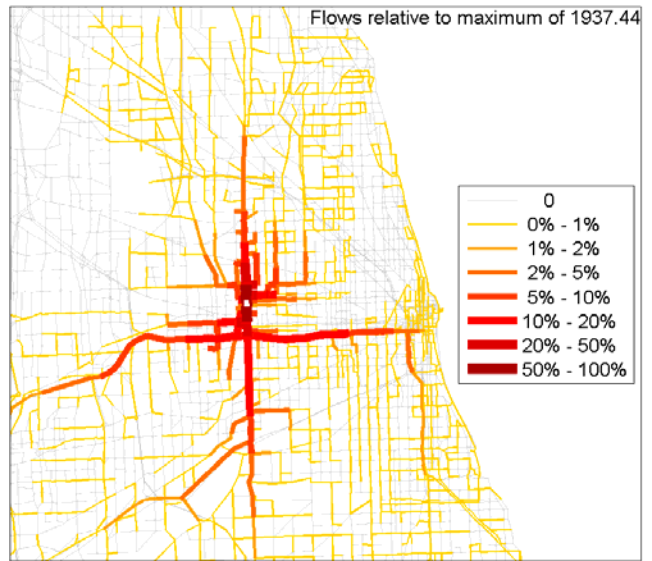
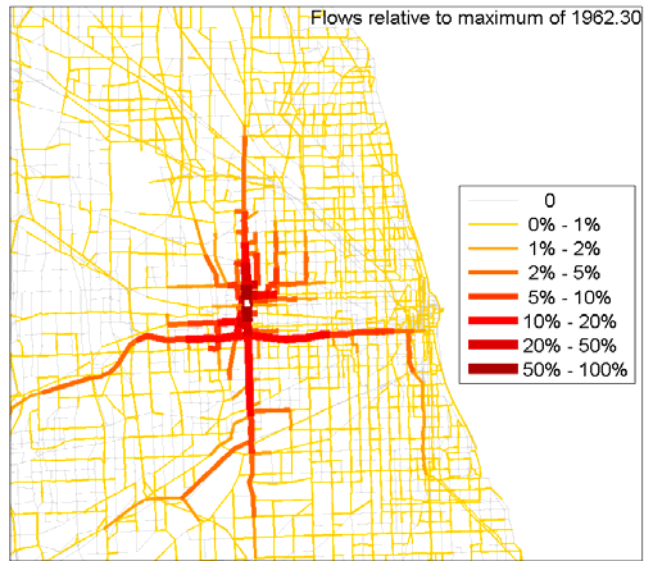


Figure 5.11 Harlem Avenue southbound link map

c) TAPAS



d) Link-based assignment: CUBE



e) Route-based assignment: VISUM

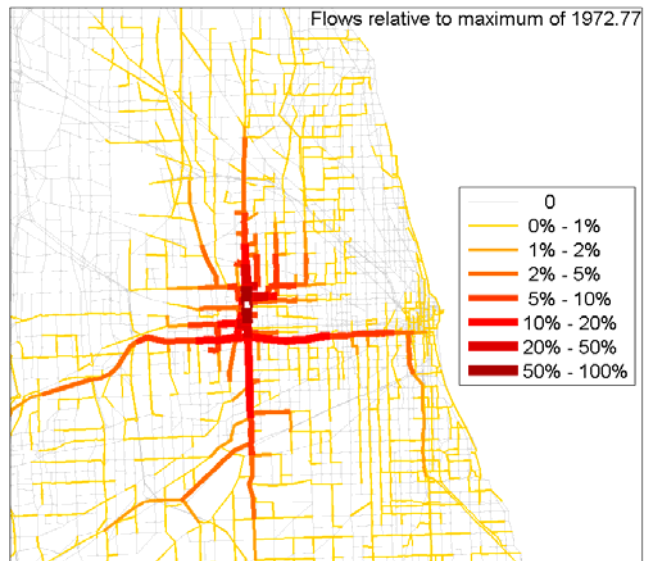


Figure 5.12 North Avenue PAS OD car flow

a) Link-based assignment: CUBE

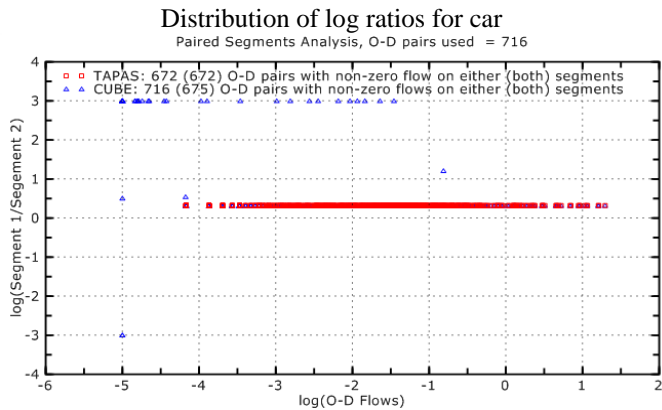
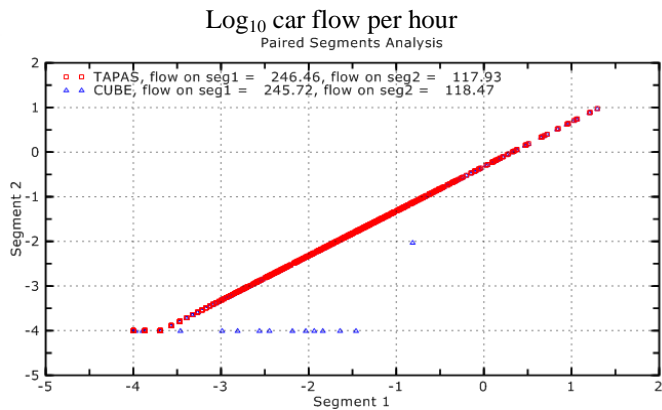
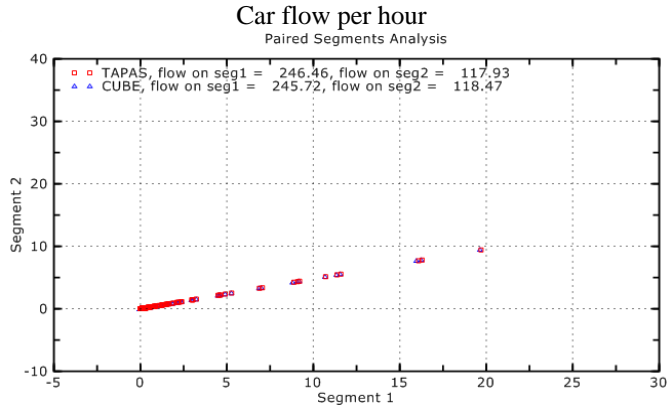


Figure 5.12 North Avenue PAS OD car flow

b) Route-based assignment: VISUM

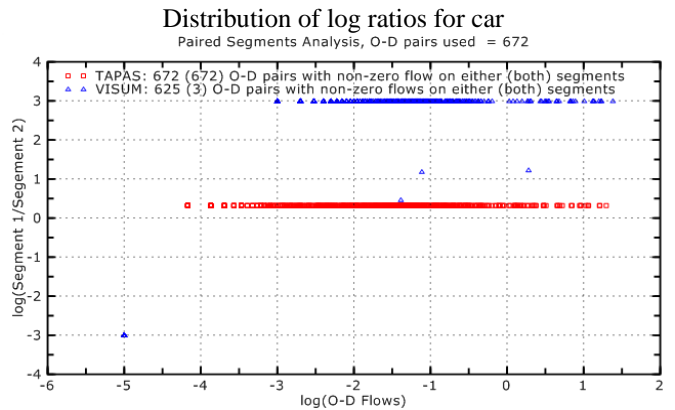
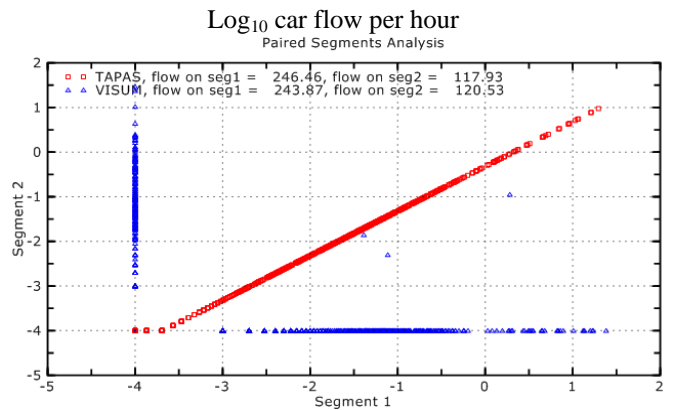
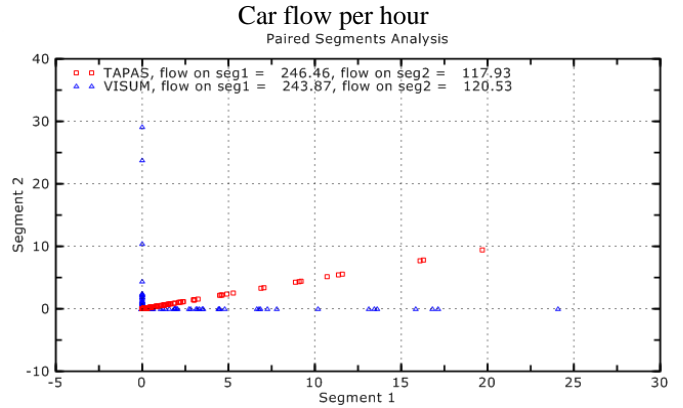


Figure 5.13 North Avenue PAS OD truck flow

a) Link-based assignment: CUBE

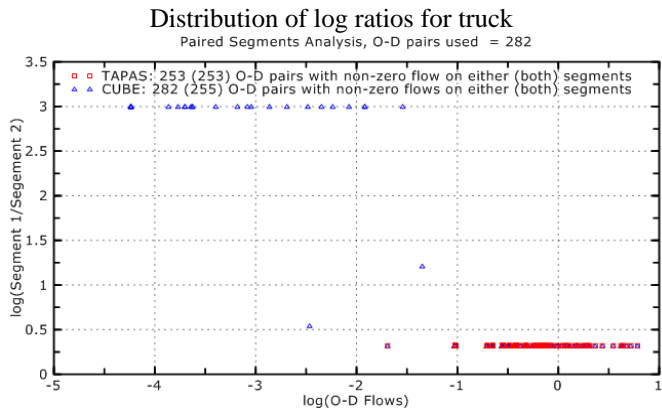
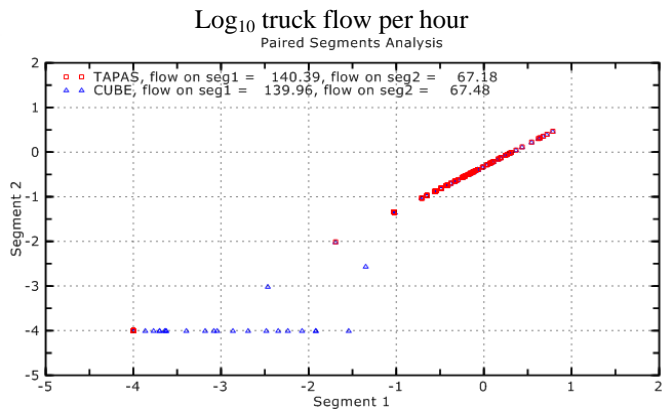
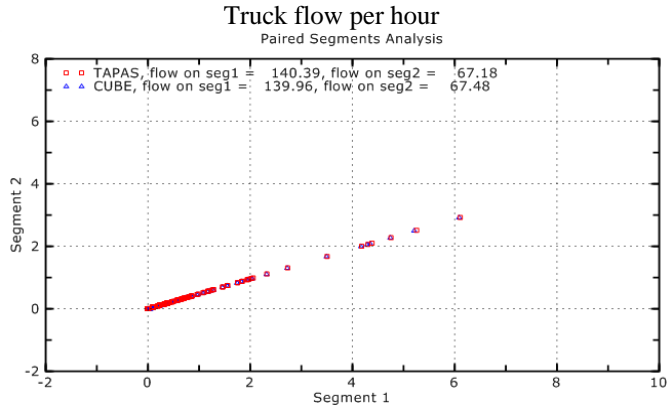


Figure 5.13 North Avenue PAS OD truck flow

b) Route-based assignment: VISUM

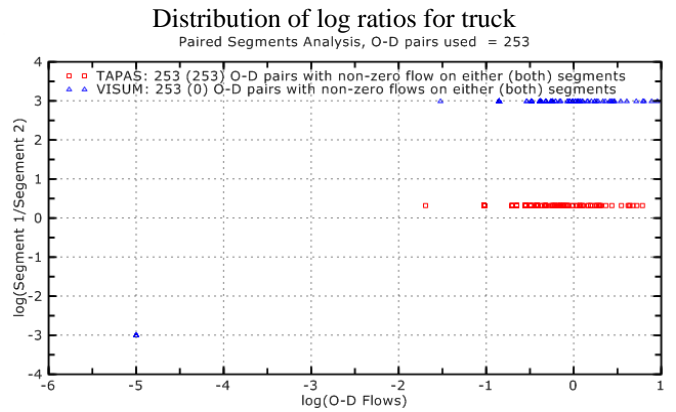
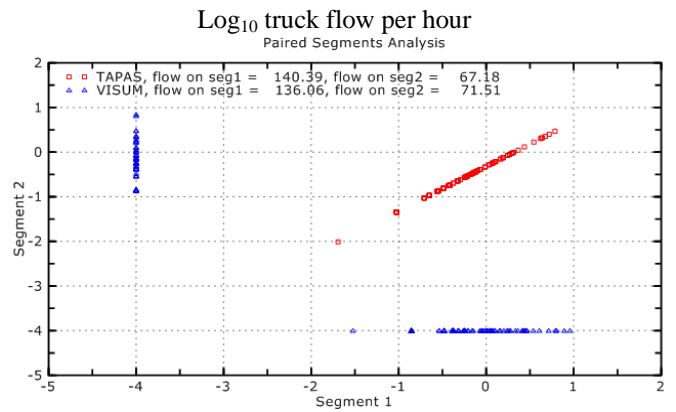
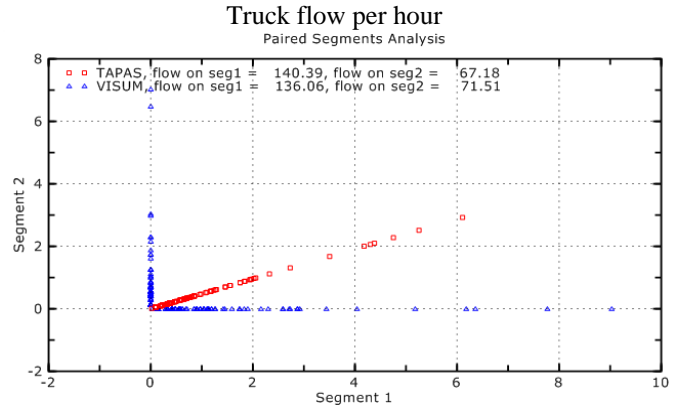
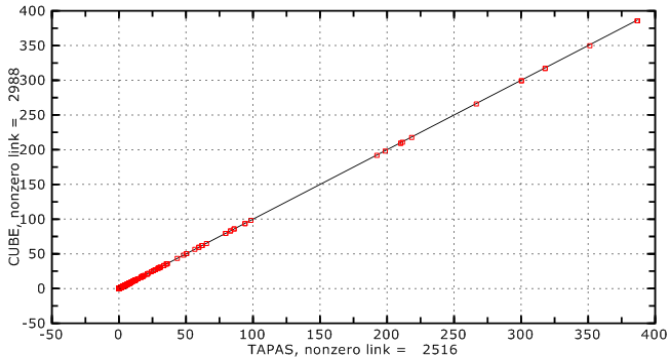


Figure 5.14 North Avenue PAS total link flow

a) Link-based assignment: CUBE

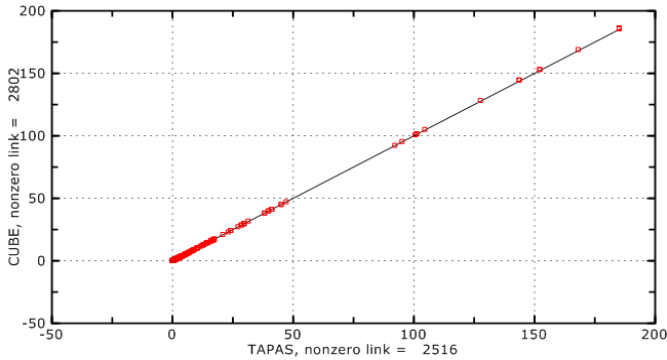
Car and truck flow per hour: segment 1

SLA Link Flow Analysis, RSME 0.093569, total nonzero links = 3000



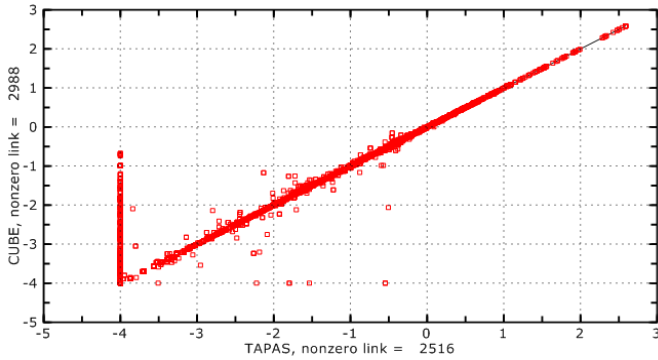
Car and truck flow per hour: segment 2

SLA Link Flow Analysis, RSME 0.075622, total nonzero links = 2812



Log₁₀ (car and truck) flow per hour: segment 1

SLA Link Flow Analysis, RSME 0.093569, total nonzero links = 3000



Log₁₀ (car and truck) flow per hour: segment 2

SLA Link Flow Analysis, RSME 0.075622, total nonzero links = 2812

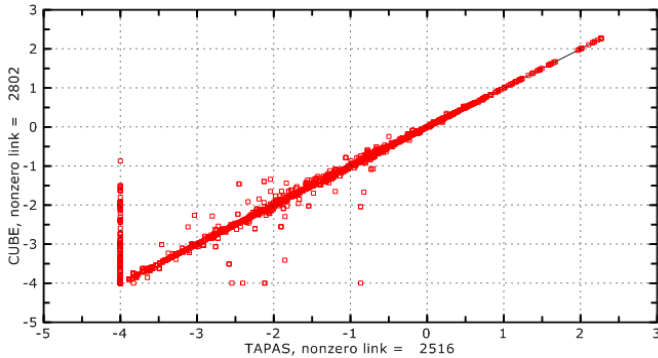
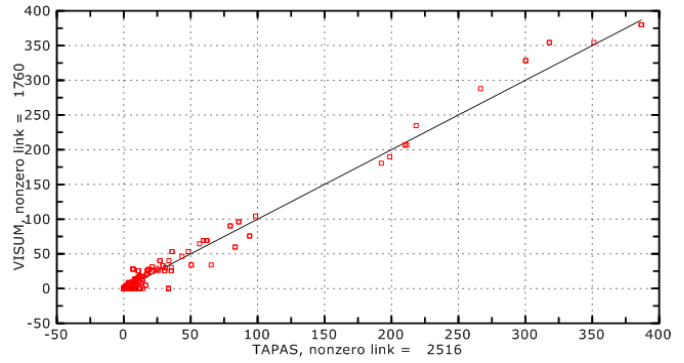


Figure 5.15 North Avenue PAS total link flow

b) Route-based assignment: VISUM

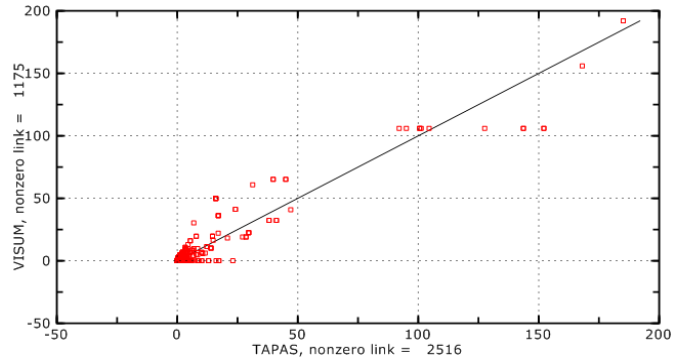
Car and truck flow per hour: segment 1

SLA Link Flow Analysis, RSME 3.098150, total nonzero links = 2519



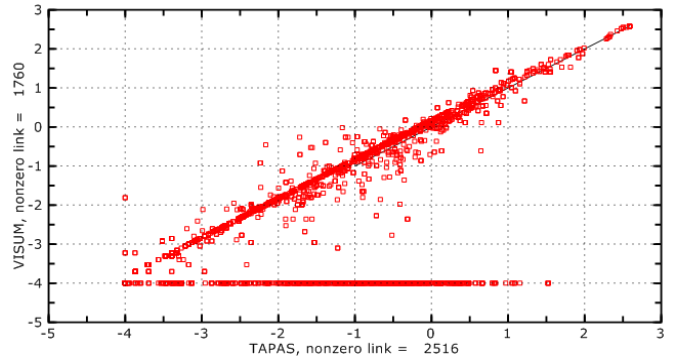
Car and truck flow per hour: segment 2

SLA Link Flow Analysis, RSME 3.206621, total nonzero links = 2516



Log₁₀ (car and truck) flow per hour: segment 1

SLA Link Flow Analysis, RSME 3.098150, total nonzero links = 2519



Log₁₀ (car and truck) flow per hour: segment 2

SLA Link Flow Analysis, RSME 3.206621, total nonzero links = 2516

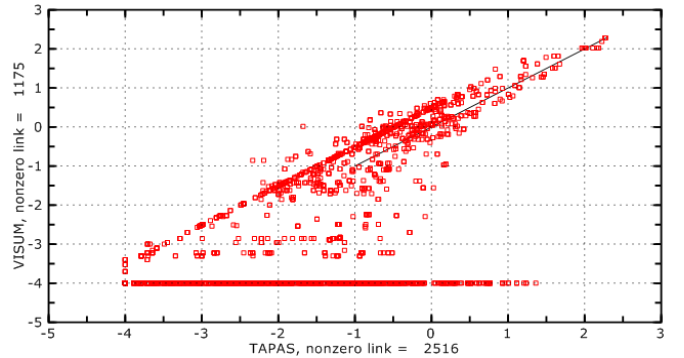
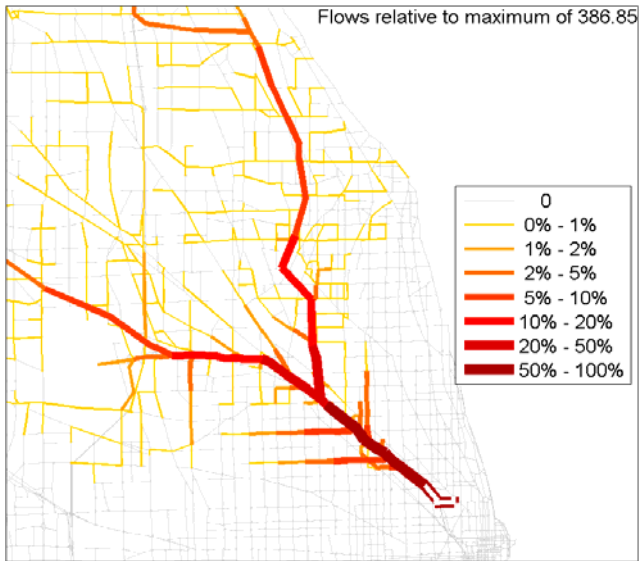
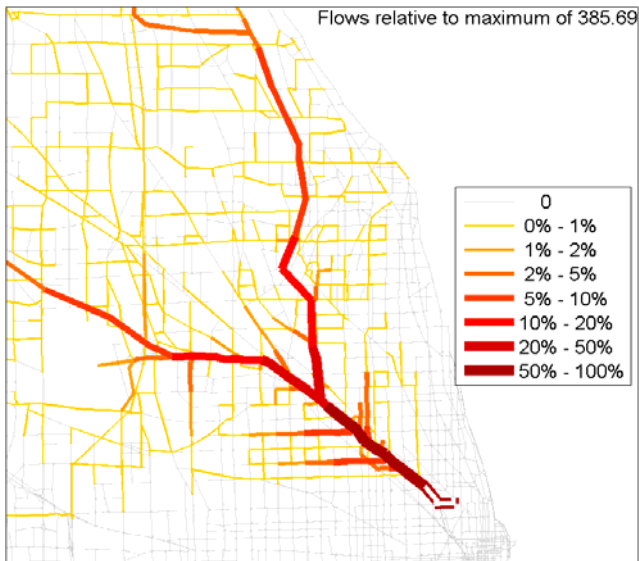


Figure 5.14 North Avenue PAS link map

c) Segment 1: TAPAS



d) Segment 1: CUBE



e) Segment 1: VISUM

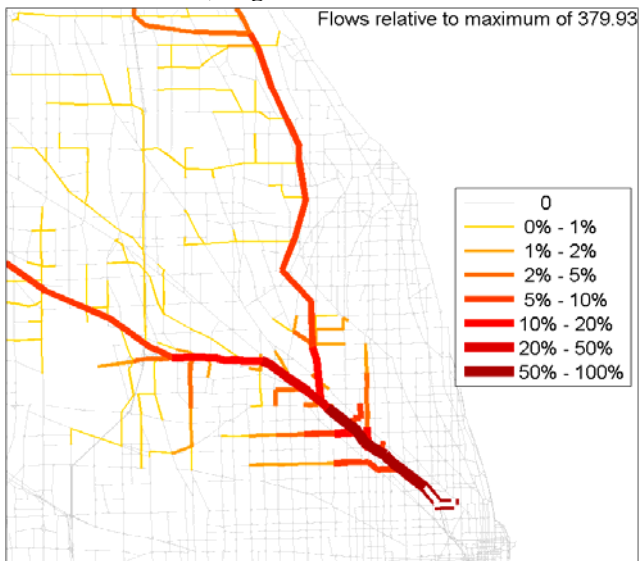
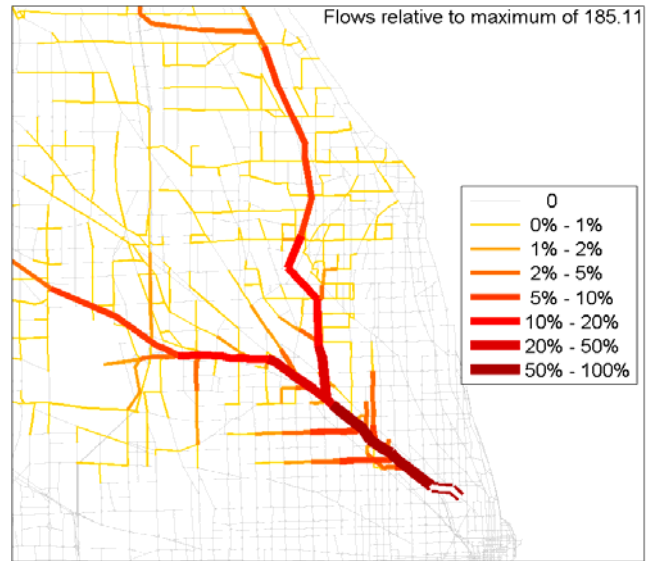
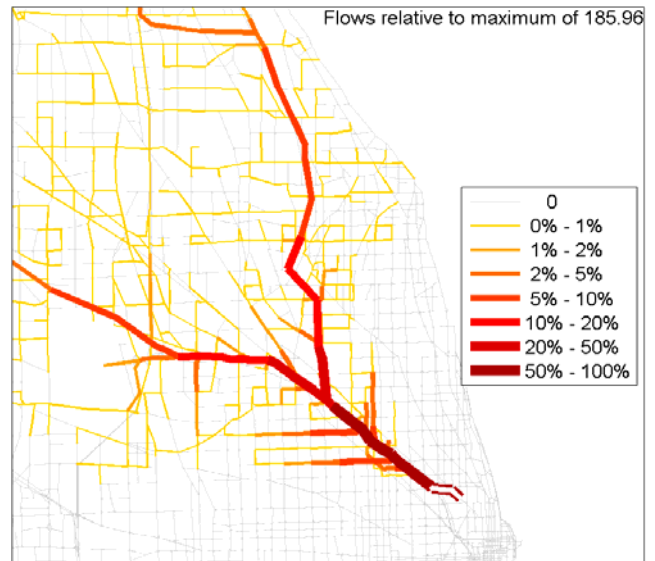


Figure 5.15 North Avenue PAS link map

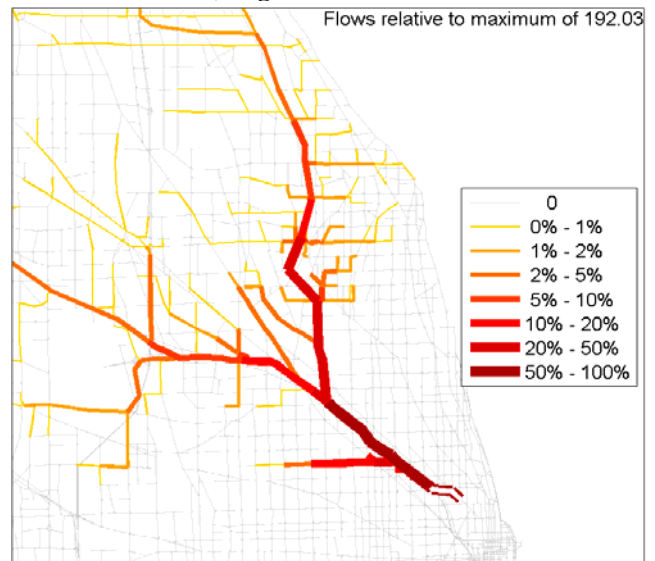
c) Segment 2: TAPAS



d) Segment 2: CUBE



e) Segment 2: VISUM



5.3.4 Discussion of two-class analysis of a pair of alternative segments

The two-class PAS analyses show the link-based method provides OD flows by segment that are almost identical to the proportions found by TAPAS for cars and trucks. The route-based method, as in the one-class case, yields a very different solution, with OD flows by class primarily assigned to one segment or the other, and not proportionally to both, as implied by the assumption of proportionality that flows between all OD pairs behave identically over the PAS. The link level flow plots also show that the link-based method provides a somewhat better result, as indicated by the scatter of the plots and the RSME. The link maps illustrate the greater extent of the link-based solutions, and the lesser and quite different location of the route-based solutions.

To examine whether the solutions satisfy the assumption of proportionality at the aggregate level of all OD pairs, the total flows by class and segment (shown near the top of the above plots) were converted to percentages for the solutions produced by CUBE, VISUM and TAPAS. The results of this analysis are shown in Table 5.2. As can be seen, the percentages of segment flow by class are equal for cars and trucks for both TAPAS and CUBE, although the values differ slightly for the two solutions; for VISUM, the percentages are somewhat different by class, and also different from the solution found by TAPAS based on the assumption of proportionality. These findings show that VISUM does not satisfy the assumption of proportionality at the aggregate level, although the differences are small.

Table 5.2 Percentage of vehicle flow using each segment by class

<u>Class</u>	<u>Segment</u>	<u>TAPAS</u>	<u>CUBE</u>	<u>VISUM</u>
Car	1 – with Bridge	67.6	67.5	66.9
	2 – without Bridge	32.4	32.5	33.1
Truck	1 – with Bridge	67.6	67.5	65.5
	2 – without Bridge	32.4	32.5	34.5

6. Recommended Future Studies

The present study has explored the characteristics of route flows and multiple-class link flows for five assignment tools and seven networks. Nevertheless, only a few links and pairs of alternative segments have been examined. Much more analysis remains before general conclusions can be drawn, and recommendations for practice offered.

Future investigations could follow four main directions:

1. additional evaluation of the differences between route flow solutions in other types of analyses or in additional case studies with different features such as turning penalties, non-BPR volume-delay functions, etc.;
2. investigation of alternative approaches to satisfying route flow and multiple-class link flow proportionality in UE assignment;
3. studies of the structure of route sets and pairs of equal-cost alternative segments in practitioner networks;
4. empirical studies to test the validity of the assumption of proportionality in reality.

Item 1 could be accomplished through ongoing collaborations with practitioners as well as by students in transportation engineering graduate programs. Items 2 and 3 are more advanced research topics. Item 4 requires more data than are presently available, but should be considered in funding future data collection efforts.

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